Notes on Mathematical Education in Leningrad (St. Petersburg)

Special schools and forms, Math programs, Math tournaments

Olympiads

Math circles

Math camps

Special schools and forms

“Big three”: 239, 30, 45 (boarding phys.-math. school of the State University)

Boarding phys.-math. schools in Soviet Union – since 1962 (Novosibirsk, Moscow, Leningrad, Kiev), professors and PhD students of the Universities teaching, selection through olympiads and special exams

Special schools: pride and controversy

The movie “Timetable for the day after tomorrow”, grasped important things: high motivation, very equal terms between teachers and students, the “learning via research” idea

Math battles of the “big three”.

High freedom in teaching
(“Abel–Ruffini theorem in problems and solutions”: groups, complex number, Riemann surfaces, Galois groups)

Math programs in special schools

“Algebra” textbooks, series editor N.Ya. Vilenkin

Fractions
Polynomials
Number Theory
Real numbers, Sets theory
Inequalities, Estimates
Quadratic equations, Vieta theorem
Systems of equations and inequalities

“Algebra and Number Theory for Math Schools” by N.B. Alfutova and A.V. Ustinov

Induction
Combinatorics (up to Catalan numbers)
Euclid’s algorithm, Fundamental Theorem of Arithmetic
Continued fractions
Congruences, Little Fermat’s and Euler’s, Chinese remainder
Rational and real numbers
Polynomials
Complex numbers
Complex numbers and geometry
Equations and systems, inequalities
Sequences and series, generating functions, Gaussian polynomials

Math programs in special schools

“Geometry” textbooks, series editor I.F. Sharygin

Main properties of plane
Triangle and Circle
Geometric problems and approaches to solving them
Angles
Similarity
Relations in triangles and circles
Problems and theorems of geometry
Areas of polygons
Circle: length and area
Coordinates and vectors
Plane transformations and isometries

P1. 20 identical balls: two chains of 4 balls and two “rectangles” 2 x 3. Compose a triangular pyramid out of those.

P2. Two intersecting lines are drawn on a piece of paper, but there’s a big hole around the intersection point. Find a way to measure the angle between the lines.

P3. Design a room of such a shape that there’s a point in the room from which none of the walls are entirely visible.

The books are full of “constructive” and non-standard problems, appealing to imagination.

Leningrad Math Olympiads (LMO)

What's special about Leningrad Olympiads:
original problems, oral form of the main two rounds, early start, each grade has its own problem set.

Roughly, we need 100 – 110 problems every year, and around 70 of them must be high-quality.

Brief history

Supervised and supported by very senior members of the "community".
Founded in 1934 by Delone, Fichtengoltz, Tartakovsky, Zhitomirsky, actively supported by Faddeev, Natanson, Krechmar, Smirnov.

First years – winners weren't allowed to take part any more, to avoid "sport", to encourage system of math enrichment.

1961 – All-Russia and 1967 – All-Soviet-Union Olympiads, support by the Ministry of Education; Leningrad and Moscow teams were direct entries to the All-Soviet-Union Olympiad.

Leningrad teams in 1980-s: 40 out of 129 1st degree diplomas at All-Soviet-Union, 21 out of 58 USSR participants at IMO.

LMO winners: Michael Gromov, Yuri Matiyasevich, Andrei Suslin, Gregory Perelman, ...

Leningrad Math Olympiads structure

School level (top six grades, Dec. – Jan.)
District level (Feb., 10,000 – 12,000 students)
City level (Feb. – March, oral, 3.5 – 4 h, 90 – 130 students in each grade)
Final, or elimination, round (March, ~ 30 students in each of the three senior grades, oral, 5 h).

Oral form:

Requires a lot of jurors (40 - 60), usually working in pairs (mistakes of jurors can't be corrected afterwards)
'+' and '– ' marks
Direct communication between students and jurors, great school of language, logic
No need to spend much time on writing solutions
Gives chances to fix mistakes on the fly

Elimination round:

Leningrad team selection and in some years also awards distribution; very high level as most of the students are from special schools or strong circles.

Leningrad Math Olympiads Jury

Jury's work:
Proposing problems and collecting those from numerous friends and colleagues
Assessing their novelty and quality (strict validation process)
Composing Olympiad problem sets
Running the Olympiads

Problem statements: high-quality language, humor. Music and lyrics.

P. The government decided to spilt and make private the state airlines company. There're 239 cities in the country, each two are connected by a line, and each line has to be sold to a private company. The parliament suspected the government in treachery and decided that for each three lines connecting some three cities, at least two of them must be sold to the same company. What could be the number of companies that bought the air lines?

Math circles

Two systems: Youth Math School of Math-Mech and circles of the Youth Creativity Palace.

Selection: through Olympiads and advertisements at schools.

Run mostly by students or PhD students of the Department of Mathematics and Mechanics of the State University. Most of them were participants of Olympiads and circles in their very recent past.

Typically high freedom and enthusiasm, “passing” the tradition and spirit. Often teaching in pairs, two sessions every week, very close personal relations with their students.
Humor in teaching: you laugh a lot in the trainings.

Extensive use of step-by-step problems in teaching (Pick’s formula, Helly’s theorem, Brouwer’s Fixed Point Theorem, etc.) even for the youngest ones.

Excellent books, rich archives.

High results in Olympiads vs. Solid foundation for math research.

Math circles: Programs

1st year:

Parity
Combinatorics
Divisibility and remainders
Pigeonhole principle
Graphs
Triangle inequality
Games (“kids enjoy playing”)
Logic, weightings, etc.

2nd year:

Induction
Combinatorics-2
Divisibility-2
Invariant
Graphs-2
Geometry
Numeration systems
Inequalities

Math circles: Programs

"Getting serious":

Induction, Peano axioms
Combinatorics, recurrences, generating functions, Catalan numbers
Number theory (Little Fermat's and Euler's, distribution of primes, arithmetical functions, algebraic structures)
Geometric transformations, group of isometries, algebraic properties of geometrical figures, transformations in coordinates
Inequalities ("means", Cauchy's, Muirhead's inequality, Jensen's inequality, norms and disks in R^n)
Graphs
Semi-invariant
Pigeonhole principle, dense subsets of R, Minkowski's lemma
Complex numbers and polynomials
Rational approximations
Elementary topology
Linear Algebra in finite-dimensional spaces
…

Math camps

Summer schools (3 – 4 weeks), winter schools (~ 1 week).

Leningrad region Summer Math School: 1970-s – 1984, 1990.

Camps of math circles.

The teaching staff is almost the same, so is the atmosphere.

Summing it up

Tradition

Large population

Enthusiasm

Problems

1. Sixty-four unit cubes on a table forming an 8 x 8 square. Is it possible to build a 4 x 4 x 4 cube out of them in such a way that any two adjacent small cubes in 8 x 8 are again adjacent in 4 x 4 x 4?

2. Two pawns, white and black, are on the chessboard, can move 1 field along horizontals and verticals at a time, in any order. Is it possible to move them so that all the mutual positions of the two pawns will occur and exactly once?

3. Several circles are cut from a plane, no circle lies within any other circle, some circles have their interiors overlapping. Prove that it's impossible to assemble the cut pieces without overlapping in such a way that they form several disjoint disks.

4. Sequence $\{a_i\}_{i=1}$ is such that $a_i <= 1988$ for all i, $a_{m+n} \mid (a_m + a_n)$. Prove that the sequence is periodic.

5. A square is cut into rectangles. We know that any horizontal line, not passing through the sides of the rectangles, intersects exactly n of them, and each vertical line, not passing through the sides of the rectangles, intersects exactly m of them. What can be the smallest possible number of the rectangles?