## MATHEMATICS COMPETITION FOR THE SEVENTH GRADERS OF HELSINKI 2012/1/18

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.
- (1) Compute  $6 \cdot 5 \cdot 4 5 \cdot 4 \cdot 3 + 4 \cdot 3 \cdot 2 3 \cdot 2 \cdot 1$ .

(2) Compute the circumference of the following figure.



(3) The base of an isosceles triangle is 5 and its area is 45. What is its height?

**a)** 4.5 **b)** 18 **c)** 9 **d)** 112.5

- (4) The sum of three consecutive integers is 42. What is the middle one?
  - **a**) 13 **b**) 14 **c**) 15 **d**) 16.
- (5) The midpoints of neighboring sides of a 1 m × 1 m-square have been connected with line segments, and we have thereby obtained a smaller square inside the original one. What is the area of the smaller square?



(6) In order to build a small forest cabin, we need one hundred logs, each of which must be five meters long. In the beginning, we have only logs which are twenty meters long each. What is the smallest number of times we have to saw through a log in order to obtain the five meter logs we need?

- (7) What is  $1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7$ ?
  - **a)** 120 **b)** 720 **c)** 5040 **d)** 40320

(8) Jack is climbing a beanstalk. The beanstalk is 88 m long. Each time Jack has climbed five meters, an ogre shakes the beanstalk and Jack slides down one meter. When Jack has reached the top, the ogre can no more shake him down. How many meters does Jack climb altogether?

**a)** 88 m **b)** 100 m **c)** 109 m **d)** 110 m

- (9) A concert is organized in the Hartwall Arena. The organizers estimate that if the price of a ticket is x euros, then the fans will buy  $10000 + 400x 10x^2$  tickets. The organizers have to choose between two prices: 30 euros per ticket and 40 euros per ticket. Which choice brings more people to the concert? Which choice earns more money for the organizers?
  - a) 30 euros brings more people and more money.
  - b) 30 euros brings more people and 40 euros brings more money.
  - c) 40 euros brings more people and 30 euros brings more money.
  - d) 40 euros brings more people and more money.
- (10) Ville went into a five-storey building in order to sell chocolate eggs. The resident of the topmost floor bought half of the eggs and a half of an egg on top of that. The resident of the fourth floor bought half of the remaining eggs and a half of an egg on top of that. The resident of the third floor, the resident of the second floor and the resident of the first floor each did the same thing. After all this Ville noticed that he had sold all of his eggs. How many chocolate eggs did Ville have in the beginning?

(11) Let  $X = 1 + 2 + 3 + 4 + \dots + 70$ . How large is X?

(12) Let us consider the number  $N = 11 \cdot 11 \cdot \ldots \cdot 11$ , where there are 2012 multiplicands. What are the last two digits of N?

- (13) What is  $\frac{789}{999} \frac{12}{99}$ ? **a)** 0.668688... **b)** 0.666... **c)** 0.668577... **d)** 0.668668...
- (14) Let  $X = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \frac{1}{3125} + \frac{1}{15625}$ . What can we say about X? **a)**  $0 < X \leq \frac{1}{4}$  **b)**  $\frac{1}{4} < X \leq \frac{1}{2}$  **c)**  $\frac{1}{2} < X \leq \frac{3}{4}$  **d)**  $\frac{3}{4} < X \leq 1$
- (15) A region of the shape of a circle has area equal to 80. As in the following picture, we remove from it two regions of the shape of a circle, the other having diagonal equal to one fourth and the other having diagonal equal to three fourths of the diagonal of the original circle.



What is the area of the remaining region?

**a)** 20 **b)** 30 **c)** 40 **d)** 50