

MATHEMATICS COMPETITION FOR THE
SEVENTH GRADERS OF OULU 2013/2/13

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. The measures of the floor of a room are 5×4 metres, and the height of the room is 2,5 metres. One pot of paint is enough if one $4 \times 2,5$ -metre wall is to be painted. How many pots are needed if the entire room (including the walls, the floor and the ceiling) is to be painted?

- a) 8 b) $8\frac{1}{2}$ c) 9 d) $9\frac{1}{2}$ e) 10

2. In the annual budget meeting of a mathematics club it is found that the club has 27,67 € in its account, and the members of the club decide to spend it on new slide rules. The local slide rule factory sells slide rules at 2,5 € apiece, but gives a 5 € discount for every order of eight. How many slide rules can the club afford to buy?

- a) 10 b) 11 c) 12 d) 13 e) 14

3. The sum of three consecutive even numbers is 144. What is the middle one?

- a) 24 b) 46 c) 48 d) 50 e) There are no such numbers.

4. In the wall of a kitchen, there is an area the shape of a 2×5 -rectangle, which will be tiled using tiles having the shape of a 2×1 -rectangle. How many different ways of doing the tiling are there?

- a) five b) six c) seven d) eight e) nine

5. What is the fifth last digit of the number $1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot 100$?

- a) 0 b) 1 c) 2 d) 3 e) 4

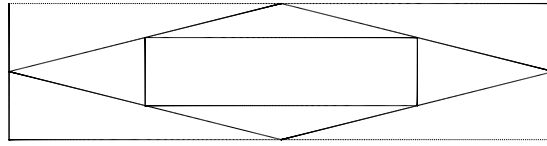
6. The bottom of a pot of paint is circular, and its diameter is 20 cm. The height of a pot is 20 cm. A box has the measures 21 cm \times 100 cm \times 39 cm. How many pots of paint fit inside the box?

- a) at most 5 b) 6 c) 7 d) 8 e) at least 9

7. How many positive integers are there which are divisible by the sum of their digits?

- a) none b) 9 c) 10 d) 42 e) infinitely many

8. The midpoints of the sides of a rectangle are joined using line segments, and a smaller quadrilateral is formed. Next, the midpoints of the sides of this quadrilateral are joined using line segments, and an even smaller rectangle is formed:



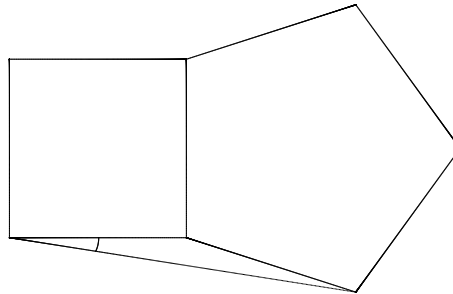
How many percents is the area of the smallest rectangle of the area of the largest rectangle?

- a) 20% b) 25% c) 30% d) 35% e) 40%

9. Let us introduce a new kind of number i , for which we agree that $i^2 = -1$. What is then $(1 + i)(1 + 2i)(1 + 3i)$?

- a) $8i$ b) 10 c) -10 d) 8 e) -8

10. In the following picture, there is a square and a regular pentagon with a common side:



How large is the angle marked in the picture?

- a) 5° b) 6° c) 7° d) 8° e) 9°

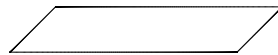
11. The numbers

$$0^2, 1^2, 2^2, 3^2, 4^2, \dots, \text{ i.e. } 0, 1, 4, 9, 16, \dots,$$

are called square numbers. When a square number is divided by eight, what are the possible remainders?

- a) 0 and 1 b) 0, 1 and 4 c) 0, 1 and 5 d) 0, 1, 4 and 5 e) 0, 1, 2, 3, 4, 5, 6 and 7

12. A parallelogram is a quadrilateral whose opposite sides are parallel. An example of a parallelogram:



Which of the following statements is true for some, but not all, parallelograms?

- a) The diagonals bisect (i.e. cut in half) each other.
 b) Each side is as long as the opposite parallel side.
 c) The area of the quadrilateral formed by joining the midpoints of neighboring sides is half of the area of the original parallelogram.
 d) The sum of the angles is 360° .
 e) The diagonals intersect each other perpendicularly.

13. How many pairs of integers x and y are there for which $x^2 + y^2 \leq 25$?

- a) none d) more than 200, but less than one thousand
 b) at least one, but at most ten e) over one thousand
 c) more than ten, but at most 200