Mathematics Competition for the Seventh Graders of Helsinki 2014/1/22

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $123 \cdot 45$.

a) 4000 **b)** 4525 **c)** 4535 **d)** 5525 **e)** 5535

2. The contents of one pot of paint is enough for painting a $2\,\text{m} \times 3\,\text{m}$-shaped area. The walls, ceiling and floor of a kitchen are to be painted. The height of the kitchen is 2,5 m, and the measures of the floor are $4\,\text{m} \times 5\,\text{m}$. The door to the kitchen ($1\,\text{m} \times 2\,\text{m}$) is not painted. The window ($1\,\text{m} \times 1\,\text{m}$) is also not painted. The paint has to be bought in full pots. How many pots of paint must be bought so that the kitchen can be painted?

a) 11 **b)** 12 **c)** 13 **d)** 14 **e)** 15

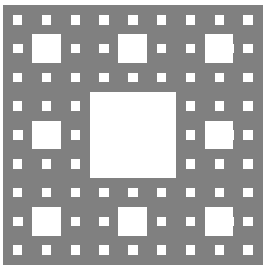
3. Compute $2 - 4 + 6 - 8 + 10 - 12 + \cdots + 98 - 100$.

a) -50 **b)** -2 **c)** -1 **d)** 0 **e)** 50

4. In an island, there are 200 inhabitants. Some of the inhabitants consume 2 kg and the rest 1 kg of tea annually per person. If the total annual consumption of tea is 300 kg of tea, how many inhabitants consume 2 kg of tea every year?

a) 0 **b)** 20 **c)** 50 **d)** 70 **e)** 100

5. A 27×27 square was divided into 9×9 squares, from which the center square was cut off. Each of the remaining squares was further subdivided into 3×3 squares of which the center square was again cut off. Finally, each of the remaining small squares was further subdivided into 1×1 squares of which the center 1×1 square was cut off. The remaining region has the following shape:



What is the area of the remaining region (i.e. the dark area in the picture), when the area of the original 27×27 square was $27 \cdot 27 = 729$?

a) 243 **b)** 444 **c)** 512 **d)** 586 **e)** 648

6. Which of the following triples of numbers a, b, c is such that there is no triangle with side lengths a, b and c?

a) $1, 2, 3$ **b)** $2, 3, 4$ **c)** $3, 4, 5$ **d)** $10, 15, 20$ **e)** $100, 100, 150$

7. Let x be some number, of which it is known that $-100 \leqslant x \leqslant 100$. What can be said with certainty?

a) $2 \cdot x$ is greater than x
b) $x \cdot x$ is greater than x
c) x is greater than the number 1
d) $x \cdot x$ is at least zero
e) $2 \cdot x$ is at most 100.

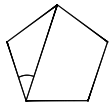
8. In a right angled triangle the length of the shortest side is 5, and the length of the longest side is 13. How long is the third side?

a) 11 **b)** $\sqrt{124}$ **c)** $\sqrt{134}$ **d)** 12 **e)** $\sqrt{154}$

9. William and Charlie are playing a game. In the game the boys take turns and try to knock down pins by rolling a ball towards them. There are four pins in a row. The boys are already so experienced in the game that they can certainly knock down any single pin or any two pins next to each other as they choose. In the game a player, who can not knock down a pin in his turn, loses. Thus, whoever knocks down the last pin wins automatically. It is William's turn to begin the game. Which pin or pins should William knock down in his opening throw in order to win?

a) One of the two center pins,
b) A pin at either end of the row,
c) Both of the two center pins; or
d) Two pins at either end of the row?

10. In the following picture, there is a regular pentagon.



How large is the angle marked in the picture?

a) 34° **b)** 35° **c)** 36° **d)** 37° **e)** 38°

11. A bicyclist decides to drive every other kilometer at the speed of 30 km/h and every other kilometer at the speed of 20 km/h. How many kilometers does he travel in an hour?

a) 18 km **b)** 24 km **c)** 25 km **d)** 26 km **e)** 28 km

12. Two positive integers a and b are such that

$$\frac{a+b}{2a+3b} = \frac{3}{8}.$$

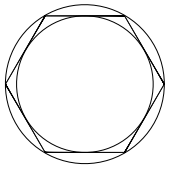
What then must be $\frac{a}{b}$?

a) $\frac{1}{6}$ **b)** $\frac{1}{5}$ **c)** $\frac{1}{4}$ **d)** $\frac{1}{3}$ **e)** $\frac{1}{2}$

13. Which of the following sums of two fractions is the largest?

a) $\frac{1}{11} + \frac{1}{19}$ **b)** $\frac{1}{12} + \frac{1}{18}$ **c)** $\frac{1}{13} + \frac{1}{17}$ **d)** $\frac{1}{14} + \frac{1}{16}$ **e)** $\frac{1}{15} + \frac{1}{15}$

14. In the following picture, there is a circle inside which there is a regular hexagon with vertices on the circle. Furthermore, a circle lies inside the hexagon so that it is tangent to each of the sides of the hexagon.



What is the ratio of the areas of the larger and the smaller circle?

a) $\frac{6}{5}$ **b)** $\frac{5}{4}$ **c)** $\frac{4}{3}$ **d)** $\frac{3}{2}$ **e)** 2