MATHEMATICS COMPETITION FOR THE
SEVENTH GRADERS OF OULU 2014/2/4

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $\frac{2}{7} - \frac{1}{8}$.

    a) $\frac{1}{8}$ **b)** $\frac{5}{28}$ **c)** $\frac{1}{6}$ **d)** $\frac{3}{16}$ **e)** $\frac{9}{56}$

2. Place the numbers 2^7, 3^5 and 5^3 in increasing order.

    a) $2^7 < 3^5 < 5^3$ **c)** $5^3 < 2^7 < 3^5$ **e)** $2^7 < 5^3 < 3^5$
    b) $3^5 < 2^7 < 5^3$ **d)** $5^3 < 3^5 < 2^7$
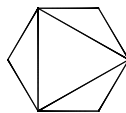
3. What is the 2014th number in the sequence 1, 2, 3, 1, 2, 3, 1, 2, 3, ...?

    a) 1   **b)** 2   **c)** 3

4. Compute $1 + 6 + 11 + 16 + \ldots + 41 + 46$, where the terms consist of those positive integers which are less than 50 and which have the remainder 1 when divided by five.

    a) 100   **b)** 225   **c)** 235   **d)** 275   **e)** 285

5. In the following picture, there is a regular hexagon with an equilateral triangle drawn inside it.

How many percents the area of the triangle is of the area of the hexagon?

    a) 50%   **b)** 55%   **c)** 60%   **d)** 65%   **e)** 70%

6. We call a positive integer "particularly nice", if it has the property, that whenever the number in question is obtained as the sum of two smaller integers, no carries are ever needed. For example, the number 13 is not particularly nice, because in the sum $9 + 4 = 13$ a carry will appear in the column of 10's. Which of the following numbers is particularly nice?

    a) 27   **b)** 38   **c)** 49   **d)** None of the three previous numbers.

7. In a room, there is a lamp and a button. When the button is pressed the lamp is lit if it was extinguished, and the the lamp is extinguished if it was lit. Every other visitor to the room presses the button twice, and every other visitor presses the button once. In the beginning, the lamp is extinguished. The first visitor to the room presses the button twice. How many times has the lamp been lit, when there has been five visitors to the room?

   **a)** 1   **b)** 2   **c)** 3   **d)** 4   **e)** 5

8. In the following picture the points A, B and C are on the arc of a circle, and O is the center of the circle.
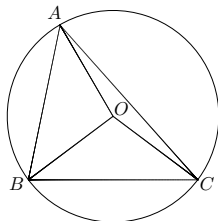
If the angle $\widehat{BAO}$ is $30°$, and the angle $\widehat{OAC}$ is $10°$, then how large is the angle $\widehat{BOC}$?

a) $70°$ **b)** $80°$ **c)** $90°$ **d)** $100°$ **e)** $110°$

9. A square shaped mosaic with measures $60\,\text{cm} \times 60\,\text{cm}$ is tiled with red right-angled triangles, except for a $10\,\text{cm} \times 20\,\text{cm}$ rectangle which is tiled with blue right-angled triangles. How many blue and red tiles are there in the mosaic? The two short sides of a blue triangle are both of length $2\,\text{cm}$, the two short sides of a red triangle are of length $1\,\text{cm}$.
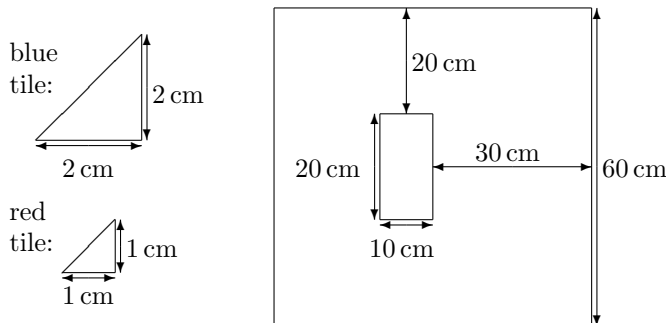
a) 50 blue ones and 3400 red ones   **d)** 100 blue ones and 6800 red ones
b) 50 blue ones and 3600 red ones   **e)** 100 blue ones and 7200 red ones
c) 50 blue ones and 7200 red ones

10. What is the last digit of the number $2014 \cdot 2014 \cdot \ldots \cdot 2014$, where the number 2014 appears 2014 times?

a) 0   **b)** 2   **c)** 4   **d)** 6   **e)** 9

11. Which of the following statements is not true for all real numbers x?

a) $x^2 + 1 \geqslant 2x$ **b)** $x^2 + 1 \geqslant -2x$ **c)** $4x^2 + 1 \geqslant 4x$
d) $x^2 + 2 \geqslant 2\sqrt{2}x$ **e)** $x^2 + 2 \geqslant 4x$

12. How many numbers in the list 1, 2, 3, ..., 999 contain the digit 7?

a) 270   **b)** 271   **c)** 280   **d)** 300   **e)** 301

13. At the start of seventh grade, the students noticed that they came from many different primary school classes. On the first day at school it was noted that each of the boys in the class room knew 4 of the girls beforehand. In the same vein, each of the girls knew 3 of the boys in the class. It was also observed that there were 12 girls in the class, but the bell rang before the number of boys had been determined. Here knowing someone is always symmetric; for instance, if Mary knows William, then William also knows Mary. How many boys were there in the class?

a) 9   **b)** 10   **c)** 11   **d)** 12
e) The number of boys cannot be determined with the given information.