# Final round - March 15th 2014

**1.** The lord of a mansion has been murdered in his study. Here is the floor plan of the second floor of the mansion. There are 15 rooms in a $3 \times 5$ grid. The study is in a corner room and in the centermost room there is a fireplace. The ghost of the lord doesn't know who the murderer is, but he knows that the murderer lives in one of the 13 other rooms (excluding the study and the middle room). The ghost wants to haunt each room exactly once (starting from the study) and then exit through the chimney. The ghost can walk through walls but not through corners. In the picture below, you can see one possible route. As you can see, this route doesn't satisfy the ghost, as one of the corner rooms is left unvisited. Help the ghost find a correct path, or explain why there isn't such a path, and the ghost is doomed to haunt the mansion forever.
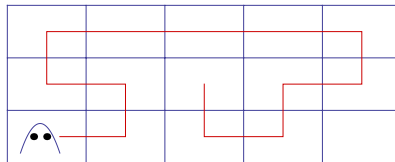


**2.** There are 15 socks in the washing machine. We notice that 12 of the socks form 6 different pairs. The other 3 have no pair, each being unique. We take two socks out of the washing machine at random. What is the probability of the two socks being a pair?

**3.** Find all positive integers $x$ and $y$, such that

$$x^3 + y = 64.$$

**4.** There are 8 collectors of ice hockey cards in the class. Each student has 8 of their own favourite card, but they agree that the other students' cards are good as well. So, they decide to trade cards during lunch break. Unfortunately, the break is short, so each student only has time to make three trades. How should the trades be arranged so that after the break each collector would have 8 different cards? [In each trade you may trade as many cards as you wish for exactly the same amount of cards, with just one person.]
Hint: You can first think of a similar situation with only 4 collectors with 4 cards each, and time for only two trades per person.

**5.** A square has area $A$. We draw a new square, whose sides are only half the length of the sides of the first square. Then we draw another square with sides one third of the sides of the second square. We continue like this, until we have a square with sides one sixth of the sides of the previous square. What is the area of the last square?