Mathematics Competition for the Seventh Graders of Turku 2014/1/22

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $3 \cdot 21 - 12 \cdot 3$.

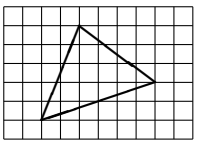
a) 27 **b)** 28 **c)** 29 **d)** 30 **e)** 31

2. The mathematics club of Peräkylä has two ways of funding their activities: selling bags of pastries and selling slide rules. The slide rules cost seven euros each and bags of pastries five euros each. The club does not need to pay anything to get the merchandise since the slide rules are old surplus and the pastries are baked by the members of the club. The notes of the financial officer of the club are in a state of chaos and he only knows that there are 37 euros in the coffers.

What has been sold?

a) Can not be determined with the given information.
b) At least eight slide rules.
c) At most two bags of pastries.
d) The financial officer has made a mistake as 37 euros can not be correct.
e) One slide rule and six bags of pastries.

3. A triangle is drawn on a squared paper:



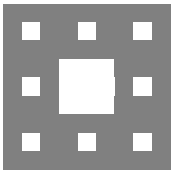
How many squares is the area of the triangle?

a) 11 **b)** 11,5 **c)** 12 **d)** 12,5 **e)** 13

4. The length of the seconds hand of a clock is 1 cm. What distance does the tip of the seconds hand travel in an hour? [The circumference of a circle is π times its diameter, and the number π is approximately 3,14.]

a) 1,8 m **b)** 1,9 m **c)** 3,6 m **d)** 3,8 m **e)** 4,8 m

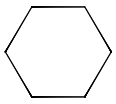
5. A 9×9 square is divided into 3×3 squares of which the middle one is cut away. The remaining 3×3 squares were divided further into 1×1 squares of which the middle one was always cut away. What remains is the following region with many holes.



What is the sum of the lengths of the boundaries between white and dark regions?

a) 43 cm **b)** 56 cm **c)** 68 cm **d)** 80 cm **e)** 92 cm

6. Each of the sides of an equilateral triangle has the length 3, and we cut away from each of its corners a piece having the shape of an equilateral triangle with each side having length 1. What remains of the original triangle is the following shape:



How much of the area of the original triangle remains?

a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** $\frac{3}{4}$ **d)** $\frac{4}{5}$ **e)** $\frac{5}{6}$

7. What is the 2014th digit after the decimal point in the decimal representation of 1/41?

a) 0 **b)** 2 **c)** 4 **d)** 3 **e)** 9

8. Compute

$$-1 \cdot 2 + 2 \cdot 3 - 3 \cdot 4 + 4 \cdot 5 - \cdots - 47 \cdot 48 + 48 \cdot 49.$$

a) 0 **b)** 50 **c)** 1200 **d)** 5350 **e)** 10000

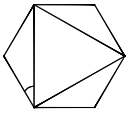
9. A positive number x satisfies

$$\left((x^2+1)^2+1\right)^2+1 = 26.$$

What number x is?

a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** 1 **d)** $\frac{3}{2}$ **e)** 2

10. In the following picture, there is a regular hexagon with an equilateral triangle drawn inside it.



How large is the angle marked in the picture?

a) 20° **b)** 25° **c)** 30° **d)** 35° **e)** 40°

11. The country of Molvania has a somewhat eccentric monetary system. It is based on just two kinds of bank notes, those of 4 dollars and those of 5 dollars. Furthermore, the ancient customs of the country prohibit givin back change. For this reason prices such as 2 or 6 dollars can not be paid at all. What is the largest (integer) price of dollars, which can not be paid no matter how many bank notes are available?

a) 7 **b)** 9 **c)** 11 **d)** 13 **e)** 15

12. In a holy city, there are five temples, each dedicated to the worship of a god. A pilgrim arrives at the city, and he wants to sacrifice coffee to each of the five gods. At the doorway of each of the temples, there is a high priest who blesses the gifts of the pilgrims. Since our pilgrim is exceptionally pious, the blessing of the high priest always doubles the quantity of coffee the pilgrim has. After visiting each of the temples, and having the blessings from each of the high priests and having sacrificed one full pot of coffee to the god of the temple, the pilgrim has no coffee left over. How many pots of coffee did the pilgrim have with him when he first arrived at the city?

a) $\frac{3}{4}$ **b)** $\frac{7}{8}$ **c)** $\frac{15}{16}$ **d)** $\frac{31}{32}$ **e)** $\frac{63}{64}$

13. A child is passing through a corridor, the floor of which is tiled with large tiles. The child avoids stepping on the edges of the tiles and can always step on the next tile or jump over it. When the length of the corridor is 10 tiles and the child begins at the first tile, in how many ways can the child choose the tiles to step on?

a) 34 **b)** 55 **c)** 89 **d)** 512 **e)** 1024