Mathematics Competition for the Seventh Graders of Oulu sub-region, 15–19 February 2016

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $11 + 22 - 33 + 44 + 55 - 66$.

   a) 0   **b)** 11   **c)** 22   **d)** 33   **e)** 44

2. In a triangle, one angle is $27°$ and another is $50°$. Compute the third angle of the triangle.

   a) $93°$ **b)** $100°$ **c)** $103°$ **d)** $110°$ **e)** $113°$

3. The following figure is colored with two colors in such a way that each cell is colored with exactly one color, and if two cells have a common side, they must be colored with different colors. How many ways to color the figure are there?
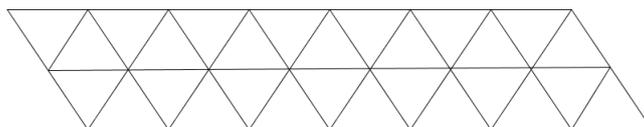
   **a)** 1   **b)** 2   **c)** 4   **d)** 8   **e)** 128

4. Compute $19 \cdot 17 - 17 \cdot 15 + 15 \cdot 13 - 13 \cdot 11$.

   a) 118   **b)** 119   **c)** 120   **d)** 121   **e)** 122

5. A sequence of numbers is said to be *arithmetic* if the difference between any two consecutive numbers is a constant. What is the fiftieth (50th) element in the arithmetic sequence 5, 66, 127, … ?

   a) 2994   **b)** 3054   **c)** 4567   **d)** 4673   **e)** 5112

6. In a cube of edge length $1\,\text{m}$, there is water up to the height $50\,\text{cm}$. This water is poured to a straight cylinder whose base is an isosceles triangle. The side lengths of the triangle are $\frac{\sqrt{5}}{2}\,\text{m}$, $\frac{\sqrt{5}}{2}\,\text{m}$ and $1\,\text{m}$. The water fills the entire cylinder but does not overflow. How tall is the cylinder?
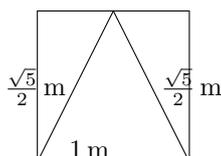
   a) $75\,\text{cm}$ **b)** $\dfrac{2}{\sqrt{5}}\,\text{m}$ **c)** $1\,\text{m}$ **d)** $\dfrac{\sqrt{5}}{2}\,\text{m}$ **e)** $1{,}5\,\text{m}$

7. A stick one meter long is divided into three parts, and the lengths of these three parts relate to each other as the numbers $2 : 5 : 7$. How long is the shortest part?

   a) $\dfrac{1}{5}\,\text{m}$ **b)** $\dfrac{2}{5}\,\text{m}$ **c)** $\dfrac{1}{7}\,\text{m}$ **d)** $\dfrac{2}{7}\,\text{m}$ **e)** $\dfrac{1}{9}\,\text{m}$

8. Let us consider the situation in the following picture, where $\alpha > 90°$. Lines l and t are parallel, and lines m and n are parallel. The angle $\angle P$ is $90°$. How large is the angle β?
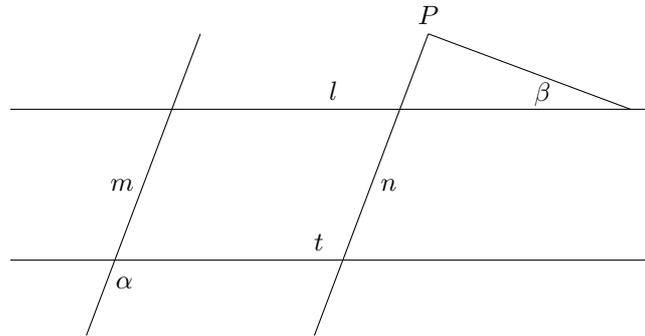
a) α **b)** $180° - \alpha$ **c)** $\alpha + 45°$ **d)** $\alpha - 45°$ **e)** $\alpha - 90°$

9. On the first row, we write the lonely number 1. On the second row we write the numbers 2, 3 and 4 so that the number 3 is right under 1. In the same vein, on the third row, we write the numbers 5, 6, 7, 8 and 9 so that the number 7 is under 3. By continuing this way, we get a figure as follows:

$$
\begin{array}{ccccc}
 & & 1 & & \\
 & 2 & 3 & 4 & \\
5 & 6 & 7 & 8 & 9 \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
$$
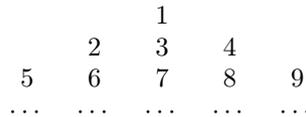
What is the leftmost number on the tenth row of the figure?

a) 81   **b)** 82   **c)** 99   **d)** 100   **e)** 101

10. Person A and person B are in an exam. A can solve each problem in 4 minutes and B in 1 minute. B takes one hour off during the exam to sleep. A and B finish the exam at the same time. How many problems did the exam have?

a) 16   **b)** 17   **c)** 18   **d)** 19   **e)** 20
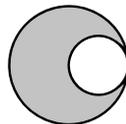
11. Order the numbers $a = 11/15$, $b = 13/19$, and $c = 16/23$ from the smallest to the largest.

a) $b < c < a$ **b)** $c < a < b$ **c)** $a < b < c$ **d)** $c < b < a$ **e)** $b < a < c$

12. What is the sum of the digits of the numbers $10^{50} - 81$?

a) 433   **b)** 442   **c)** 542   **d)** 551   **e)** 560

13. The radius of the larger circle is twice the radius of the smaller circle. How large a portion of the figure has been colored?

a) $\dfrac{3}{4}$ **b)** $\dfrac{4}{5}$ **c)** $\dfrac{5}{6}$ **d)** $\dfrac{6}{7}$ **e)** $\dfrac{7}{8}$

14. What is the remainder when the number $A = 1 + 2 + 3 + 4 + 5 + \ldots + 2016$ is divided by the number 5?

a) 0   **b)** 1   **c)** 2   **d)** 3   **e)** 4

15. How many integer solutions does the equation $x^2 + y^2 = 5$ have?

a) 4   **b)** 8   **c)** 12   **d)** 16   **e)** 20