Mathematics Competition for the Seventh Graders of Helsinki, 6–10 March, 2017

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $466 - 21$.

a) 445 **b)** 412 **c)** -412 **d)** 455 **e)** 499

2. Compute $23 \cdot 25$.

a) 565 **b)** 575 **c)** 585 **d)** 595 **e)** 605

3. We wish to build a fence around an area the shape of a rectangle. The area of the rectangle is to be $100\,\mathrm{m}^2$. Which of the following options requires the least work on fencing?

a) $5\,\mathrm{m} \times 20\,\mathrm{m}$ **b)** $10\,\mathrm{m} \times 10\,\mathrm{m}$ **c)** $1\,\mathrm{m} \times 100\,\mathrm{m}$ **d)** $25\,\mathrm{m} \times 4\,\mathrm{m}$ **e)** $2\,\mathrm{m} \times 50\,\mathrm{m}$

4. We have a bucket of 10 litres and a tub of 100 litres. Which amounts of water (in litres) can we measure using these two vehicles?

a) 1, 15 and 20 **b)** 5 and 10 **c)** 62 **d)** 20 and 60 **e)** All of the previous amounts.

5. The length of a line segment AB is 1. Two points A' and B' dissect the segment AB into three parts of equal lengths, and they lie on the segment as in the following picture, i.e. $AA' = A'B' = B'B$. A point P lies on the segment AB between the points A' and B'. Furthermore, we know that $3 \cdot A'P = PB'$. Compute AP.

a) $\frac{1}{3}$ **b)** $\frac{5}{12}$ **c)** $\frac{4}{9}$ **d)** $\frac{1}{2}$ **e)** $\frac{2}{3}$

6. At first yoghurt costs 1.00 euros per litre. After one year, when the market conditions change, the price increases 10 %, two years later the price decreases by 20 %, and three years later it increases by 50 %. How much does a litre of yoghurt cost after all this?

a) 0.77 euros **b)** 1.32 euros **c)** 1.13 euros **d)** 1.54 euros **e)** 1.98 euros

7. Let us define a new operation for numbers using the familiar addition and multiplication: $a \otimes b = 3a + 7b$. For example, $3 \otimes 2 = 23$. What is

$$(1 \otimes 2) + (3 \otimes 4)?$$

a) 51 **b)** 52 **c)** 53 **d)** 54 **e)** 55

8. How many of the numbers 2^1, 2^2, 2^3, …, 2^{100} end with the digit 6? [Here 2^N means the product $2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2$, where the number 2 appears N times.]

a) 22 **b)** 23 **c)** 24 **d)** 25 **e)** 26

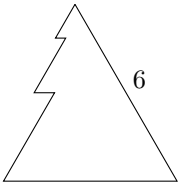
9. There are 21 children in a daycare group, and each of them speaks at least one language. It is known that five of the children speak at least Finnish and Russian, six of the children speak at least Finnish and Swedish, and three of the children speak at least Swedish and Russian. Furthermore, we know that two of the children speak Finnish, Swedish and Russian, and that none of the children speaks any other languages. How many of the children speak only one language?

a) not solvable with the given data **b)** 0 **c)** 10 **d)** 8 **e)** 11

10. Which of the following digits does not appear in the decimal representation of the number $\frac{58}{333}$?

a) 1 **b)** 4 **c)** 7 **d)** 9 **e)** All of the four previous digits appear.

11. What is the circumference (i.e. the length of the boundary) of the following figure? All the angles in it are 60° or 300°.

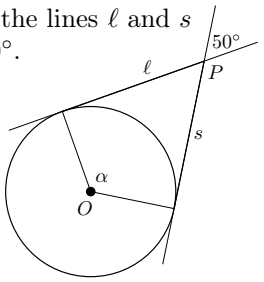


a) 15 **b)** 16 **c)** 17 **d)** 18 **e)** 19

12. Five players A, B, C, D and E participate in a chess tournament. Each player plays exactly once against each of the other players. By lunch break A has played four games, B has played three games, C has played two games and D has played one game. How many games has E played?

a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** 4

13. In the following figure, the point O is the center of the circle and the lines ℓ and s are its tangents intersecting each other at the point P at an angle of 50°.
How large is the angle α?



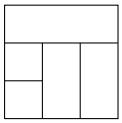
a) 100°
b) 130°
c) 155°
d) 170°
e) 200°

14. Compute $\left(1+\frac{1}{7}\right)\left(1+\frac{1}{7+1}\right)\left(1+\frac{1}{7+2}\right)\left(1+\frac{1}{7+3}\right)\left(1+\frac{1}{7+4}\right)\left(1+\frac{1}{7+5}\right)\left(1+\frac{1}{7+6}\right)\left(1+\frac{1}{7+7}\right)$.

a) $\frac{1}{7\cdot(7+1)\cdots(7+7)}$ **b)** $\frac{2}{7\cdot(7+1)\cdots(7+7)}$ **c)** $2+\frac{1}{7}$

d) $1+\frac{1}{7\cdot(7+1)\cdots(7+7)}$ **e)** $2+\frac{1}{7\cdot(7+1)\cdots(7+7)}$

15. We color each region of the following figure with one of the colors blue, red, yellow and green in such a manner that two neighbouring regions are always colored with different colors. In how many ways can this be done?



a) 84 **b)** 88 **c)** 92 **d)** 96 **e)** 100