

MATHEMATICS COMPETITION FOR THE SEVENTH
GRADERS OF OULU SUB-REGION, FINAL 13.4.2024

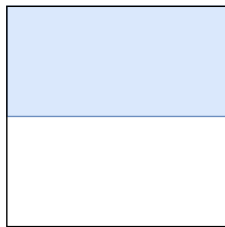
Remember to carefully justify each step of your solution!

1. We know that x and y are positive integers such that x is odd and y is even, and that the following equation holds:

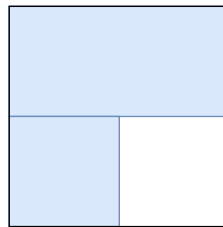
$$(x + 2)y = 24.$$

Find the numbers x and y . Are there multiple solutions? Why / why not?

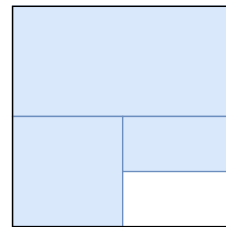
2. Consider a square of area 1. We'll start by coloring half of the square. In the next step we'll color half of the remaining area. We'll continue in the same manner. What is the area of the total coloured area after 8 steps? Provide your answer as a fraction.



Step 1



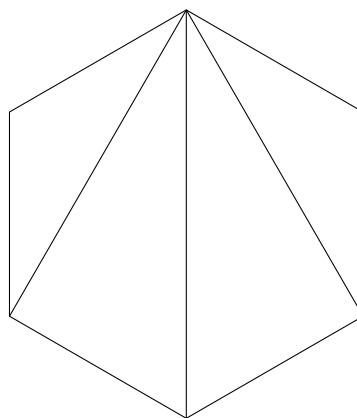
Step 2



Step 3

3. The picture shows a regular hexagon i.e. a hexagon with equal sides and equal angles. Line segments are drawn from one of its vertices to all other vertices.

You can now see four triangles in the picture. Deduce all the angles of the triangles. You can use the fact that the sum of angles in a triangle is always 180° . Use careful reasoning. Do not use an angle ruler or estimate the angles by eye.



4. If we arrange the chairs of a school's great hall into 7 rows of equal length, then 3 chairs are left over. If we instead try to arrange the chairs into 9 rows of equal length, the last row is missing 3 chairs. There are more than 50 but less than 100 chairs. How many chairs are there? Is it possible to arrange the chairs into rows of equal length?

5. Battleship is a two player game played on a square shaped grid. One player hides a number of ships in the grid. The other player then tries to guess the position of the ships by guessing squares in the grid. After every guess, the person who hid the ships will reveal if the guesser got a hit (guessed a square with a ship section in it) or a miss.

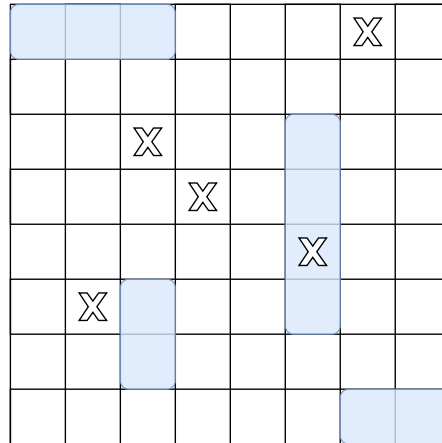


Figure 1: Usually there are several ships of different lengths hidden in the grid. The picture shows the ships as blue rectangles and guesses made by the other player as X's. A 4 square long ship has been hit.

Your opponent has hidden only one ship of length 5 in the grid. The ship can be anywhere in the grid either horizontally or vertically, but not diagonally. The rest of the grid is empty. Consider the situation on two grids of different sizes:

- a) The grid is 5×5 squares.
- b) The grid is 10×10 squares.

Invent a guessing strategy, which definitely hits the hidden ship with the smallest possible number of guesses. One hit is enough. Explain your strategy verbally or with an illustration. Justify why it is the optimal strategy.