9th Iranian Geometry Olympiad

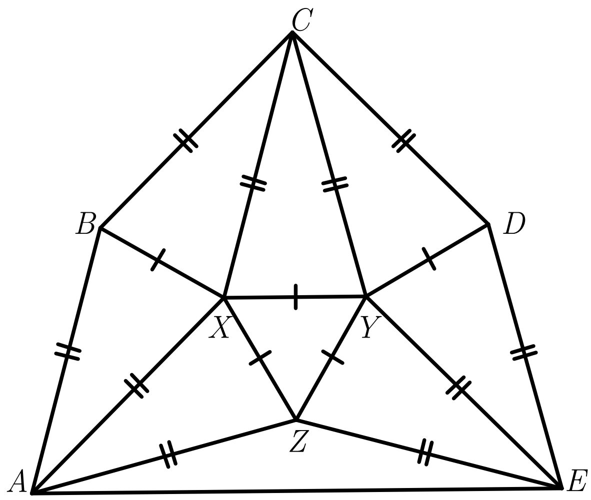
Elementary level

October 14, 2022

The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.com

---

Problem 1. Find the angles of the pentagon $ABCDE$ in the figure below.



Problem 2. An isosceles trapezoid $ABCD$ ($AB \parallel CD$) is given. Points E and F lie on the sides BC and AD, and the points M and N lie on the segment EF such that $DF = BE$ and $FM = NE$. Let K and L be the foot of perpendicular lines from M and N to AB and CD, respectively. Prove that $EKFL$ is a parallelogram.

Problem 3. Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$ and $\angle BDE = \angle EAC = 30^\circ$. Find the possible values of $\angle BEC$.

Problem 4. Let AD be the internal angle bisector of triangle ABC. The incircles of triangles ABC and ACD touch each other externally. Prove that $\angle ABC > 120^\circ$. (Recall that the incircle of a triangle is a circle inside the triangle that is tangent to its three sides.)

Problem 5. a) Do there exist four equilateral triangles in the plane such that each two have exactly one vertex in common, and every point in the plane lies on the boundary of at most two of them?

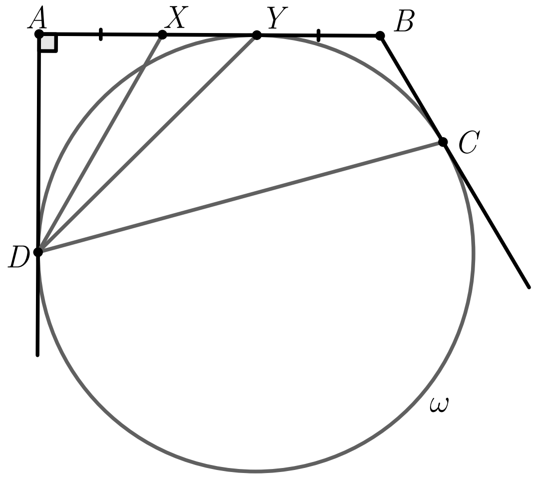
b) Do there exist four squares in the plane such that each two have exactly one vertex in common, and every point in the plane lies on the boundary of at most two of them?

(Note that in both parts, there is no assumption on the intersection of interior of polygons.)

Time: 4 hours.

Each problem is worth 8 points.

Problem 1. In the figure below we have $AX = BY$. Prove that $\angle XDA = \angle CDY$.



Problem 2. Two circles ω_1 and ω_2 with equal radius intersect at two points E and X. Arbitrary points C, D lies on ω_1, ω_2. Parallel lines to XC, XD from E intersect ω_2, ω_1 at A, B, respectively. Suppose that CD intersect ω_1, ω_2 again at P, Q, respectively. Prove that $ABPQ$ is concyclic.

Problem 3. Let O be the circumcenter of triangle ABC. Arbitrary points M and N lie on the sides AC and BC, respectively. Points P and Q lie in the same half-plane as point C with respect to the line MN, and satisfy $\triangle CMN \sim \triangle PAN \sim \triangle QMB$ (in this exact order). Prove that $OP = OQ$.

Problem 4. We call two simple polygons P, Q *compatible* if there exists a positive integer k such that each of P, Q can be partitioned into k congruent polygons similar to the other one. Prove that for every two even integers $m, n \geq 4$, there are two compatible polygons with m and n sides. (A simple polygon is a polygon that does not intersect itself.)

Problem 5. Let $ABCD$ be a quadrilateral inscribed in a circle ω with center O. Let P be the intersection of two diagonals AC and BD. Let Q be a point lying on the segment OP. Let E and F be the orthogonal projections of Q on the lines AD and BC, respectively. The points M and N lie on the circumcircle of triangle QEF such that $QM \parallel AC$ and $QN \parallel BD$. Prove that the two lines ME and NF meet on the perpendicular bisector of segment CD.

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.

9^{th} **Iranian Geometry Olympiad**

Advanced level

October 14, 2022

The problems of this contest are to be kept confidential until they are posted on the official IGO website: igo-official.com

Problem 1. Four points A, B, C, and D lie on a circle ω such that $AB = BC = CD$. The tangent line to ω at point C intersects the tangent line to ω at point A and the line AD at points K and L. The circle ω and the circumcircle of triangle KLA intersect again at M. Prove that $MA = ML$

Problem 2. We are given an acute triangle ABC with $AB \neq AC$. Let D be a point on BC such that DA is tangent to the circumcircle of triangle ABC. Let E and F be the circumcenters of triangles ABD and ACD, respectively, and let M be the midpoint of EF. Prove that the line tangent to the circumcircle of AMD through D is also tangent to the circumcircle of ABC.

Problem 3. In triangle ABC ($\angle A \neq 90°$), let O, H be the circumcenter and the foot of the altitude from A respectively. Suppose M, N are midpoints of BC, AH respectively. Let D be the intersection of AO and BC and let H' be the reflection of H about M. Suppose that the circumcircle of $OH'D$ intersects the circumcircle of BOC at E. Prove that NO and AE are concurrent on the circumcircle of BOC.

Problem 4. Let $ABCD$ be a trapezoid with $AB \parallel CD$. Its diagonals intersect at a point P. The line passing through P parallel to AB intersects AD and BC at Q and R, respectively. Exterior angle bisectors of angles DBA, DCA intersect at X. Let S be the foot of X onto BC. Prove that if quadrilaterals $ABPQ, CDQP$ are circumscribed, then $PR = PS$.

Problem 5. Let ABC be an acute triangle inscribed in a circle ω with center O. Points E, F lie on its sides AC, AB, respectively, such that O lies on EF and $BCEF$ is cyclic. Let R, S be the intersections of EF with the shorter arcs AB, AC of ω, respectively. Suppose K, L are the reflection of R about C and the reflection of S about B, respectively. Suppose that points P and Q lie on the lines BS and RC, respectively, such that PK and QL are perpendicular to BC. Prove that the circle with center P and radius PK is tangent to the circumcircle of RCE if and only if the circle with center Q and radius QL is tangent to the circumcircle of BFS.

Time: 4 hours and 30 minutes.
Each problem is worth 8 points.