Mathematics Competition for the Seventh Graders of Oulu 2012/1/18

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

(1) The sum of three consecutive integers is 42. What is the middle one?

a) 13 **b)** 14 **c)** 15 **d)** 16.

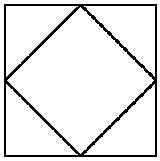
(2) The length of a side of a regular hexagon is 5. What is the length of its diagonal (from one vertex to the opposing vertex)?

a) 5 **b)** $5\sqrt{3}$ **c)** 10 **d)** $10\sqrt{3}$

(3) Compute $9 \cdot 8 - 8 \cdot 7 + 7 \cdot 6 - 6 \cdot 5$.

a) 38 **b)** -28 **c)** -38 **d)** 28

(4) The midpoints of neighboring sides of a $1\,\mathrm{m} \times 1\,\mathrm{m}$-square have been connected with line segments, and we have thereby obtained a smaller square inside the original one. What is the area of the smaller square?



a) $0.25\,\mathrm{m}^2$ **b)** $0.5\,\mathrm{m}^2$ **c)** $1\,\mathrm{m}^2$ **d)** $2\,\mathrm{m}^2$

(5) What are the last two digits of the number $25 \cdot 25 \cdot \ldots \cdot 25$?

a) 25 **b)** 35 **c)** 45 **d)** 55

(6) In order to build a small forest cabin, we need one hundred logs, each of which must be five meters long. In the beginning, we only have logs which are twenty meters long each. What is the smallest number of times we have to saw through a log in order to obtain the five meter logs we need?

a) 50 **b)** 75 **c)** 99 **d)** 100

(7) A concert is organized in the Raksila Hall. The organizers estimate that if the price of a ticket is x euros, then the fans will buy $10000 + 400x - 10x^2$ tickets. The organizers have to choose between two prices: 30 euros per ticket and 40 euros per ticket. Which choice brings more people to the concert? Which choice earns more money for the organizers?

a) 30 euros brings more people and more money.
b) 30 euros brings more people and 40 euros brings more money.
c) 40 euros brings more people and 30 euros brings more money.
d) 40 euros brings more people and more money.

(8) What is the sum of the angles of a pentagon?

a) 480° **b)** 540° **c)** 600° **d)** 720°

(9) The brothers Ibrahim and Hussein were travelling and had just camped alongside the road in order to eat a meal. Ibrahim had prepared five sandwiches and Hussein had prepared three. A stranger showed up and he was also hungry. He asked for food from the brothers and offered to pay eight gold coins for his meal. The brothers agreed and all three ate equal amount of bread. How should the brothers divide the eight gold coins among themselves so that each of them would get the same compensation for each piece of bread they gave to the stranger?

a) Four coins for both.
b) Five coins for Ibrahim and three for Hussein.
c) Six coins for Ibrahim and two for Hussein.
d) Seven coins for Ibrahim and one for Hussein.

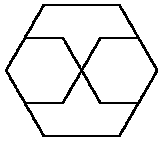
(10) Is the number $1 \cdot 3 \cdot 5 + 2 \cdot 4 \cdot 6 + 3 \cdot 5 \cdot 7 + \ldots + 2008 \cdot 2010 \cdot 2012$ even or odd?

a) It is even. **b)** It is odd.

(11) Let $X = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$. What can we say about X?

a) $0 < X \leqslant \frac{1}{4}$ **b)** $\frac{1}{4} < X \leqslant \frac{1}{2}$ **c)** $\frac{1}{2} < X \leqslant \frac{3}{4}$ **d)** $\frac{3}{4} < X \leqslant 1$

(12) The area of a region having the shape of a regular hexagon is 10. As in the following picture, we remove from it two regions, both of which are of the shape of a regular hexagon with diagonal (from one vertex to the opposing vertex) equal to half of the diagonal (from one vertex to the opposing vertex) of the original hexagon.



What is the area of the remaining region?

a) 3 **b)** 4 **c)** 5 **d)** 6

(13) How many pairs of integers x, y are there for which $1 + x^2 = y^2$?

a) 1 **b)** 2 **c)** 4 **d)** more than 4

(14) One the angles of a triangle is 72° and the difference of its other two angles is 48°. How large is its largest angle?

a) 72° **b)** 78° **c)** 82° **d)** 88°