Mathematics Competition for the Seventh Graders of Turku 2012/1/18

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

(1) Compute $20 \cdot 12 - 11 \cdot 21$.

a) -31 **b)** 0 **c)** 9 **d)** 31

(2) The product of two consecutive natural numbers is 210. How large is the smaller of the numbers?

a) 13 **b)** 14 **c)** 15 **d)** 16

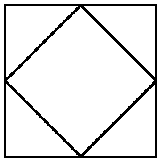
(3) How many rotations does the sweep hand [the hand showing seconds] of an ordinary clock make in one hour?

a) 1 **b)** 12 **c)** 60 **d)** 3600

(4) The area of a square is 25. What is its circumference?

a) 5 **b)** 10 **c)** 15 **d)** 20

(5) The midpoints of neighboring sides of a $1\,\mathrm{m} \times 1\,\mathrm{m}$-square have been connected with line segments, and we have thereby obtained a smaller square inside the original one. What is the area of the smaller square?

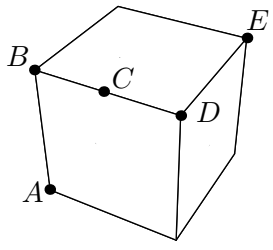


a) $0.25\,\mathrm{m}^2$ **b)** $0.5\,\mathrm{m}^2$ **c)** $1\,\mathrm{m}^2$ **d)** $2\,\mathrm{m}^2$

(6) In order to build a small forest cabin, we need one hundred logs, each of which must be five meters long. In the beginning, we only have logs which are twenty meters long each. What is the smallest number of times we have to saw through a log in order to obtain the five meter logs we need?

a) 50 **b)** 75 **c)** 99 **d)** 100

(7) An ant may crawl across the surface of a cube every which way it wants. In the beginning it is at the vertex A and it would like to get to the vertex E. Which of the points B, C and D lies on a shortest possible path?



a) The point B. **b)** The point C. **c)** The point D.

(8) A concert is organized in the Turku Hall. The organizers estimate that if the price of a ticket is x euros, then the fans will buy $10000 + 400x - 10x^2$ tickets. The organizers have to choose between two prices: 30 euros per ticket and 40 euros per ticket. Which choice brings more people to the concert? Which choice earns more money for the organizers?

a) 30 euros brings more people and more money.
b) 30 euros brings more people and 40 euros brings more money.
c) 40 euros brings more people and 30 euros brings more money.
d) 40 euros brings more people and more money.

(9) With how many zeroes does the number $1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot 30$ end?

a) 4 **b)** 5 **c)** 6 **d)** 7

(10) Let $X = 1 + 2 + 3 + \ldots + 63 + 64 + 65 + 64 + 63 + \ldots + 3 + 2 + 1$. How large is X?

a) 2015 **b)** 2080 **c)** 4180 **d)** 4225

(11) The boys living in a six-storey building wanted to find out which is the tallest fall to the lawn a bottle survives. They had two identical bottles at their disposal. They figured out that if a bottle breaks when dropped from a certain floor of the building, then it would have broken also if it had been dropped from any higher floor. The boys wanted to investigate the matter in fewest possible experiments, as the risk of the irritable janitor finding out about the research kept constantly rising. From which floor should they drop a bottle first?

a) The first floor. **b)** The third floor. **c)** The fourth floor. **d)** The sixth floor.

(12) Let $X = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \frac{1}{729}$. What can we say about X?

a) $0 < X \leqslant \frac{1}{4}$ **b)** $\frac{1}{4} < X \leqslant \frac{1}{2}$ **c)** $\frac{1}{2} < X \leqslant \frac{3}{4}$ **d)** $\frac{3}{4} < X \leqslant 1$

(13) The sum of the vertex angle and a base angle of an isosceles triangle is $112°$. How large is the vertex angle?

a) $24°$ **b)** $34°$ **c)** $44°$ **d)** $54°$

(14) Are there integers x and y so that $x^2 + 6 = y^2$?

a) Yes, there are. **b)** No, there are not.