Mathematics Competition for the
Seventh Graders of Turku 2013/2/7

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. The walls and the floor of a bathroom are waterproofed. The measures of the floor are 2×3 metres and the height of the room is 3 metres. One pot of liquid waterproofing membrane is enough for waterproofing the floor. How many pots of the substance are needed when it is only sold in full pots?

a) 4 **b)** 5 **c)** 6 **d)** 7 **e)** 8

2. A family has three small dogs, each of which eats one bag of dog food every day. In the local supermarket one bag costs $5 \in$, but due to a marketing campaign one can buy eight bags for the price of seven. How much does it cost for the family to buy dog food for two weeks?

a) $150 \in$ **b)** $165 \in$ **c)** $185 \in$ **d)** $190 \in$ **e)** $280 \in$

3. The product of three consecutive integers is 157. What is the middle one?

a) There are no such integers. **b)** 5 **c)** 7 **d)** 9 **e)** There are several right answers.

4. The legs (i.e. the two shortest sides) of a right triangle have the lengths 3 and 4. What is the length of the height drawn against the hypotenuse (the longest side)?

a) 1,8 **b)** 2 **c)** 2,2 **d)** 2,4 **e)** 2,6

5. In a list of five numbers the first number is 111 and the last one is 333, and the sum of any three consecutive numbers in the list is 999. What is the number in the middle of the list?

a) 222 **b)** 333 **c)** 444 **d)** 555 **e)** 666

6. What is the sum of the digits in the decimal representation of the product $111\,111\,111 \cdot 111\,111\,111$?

a) 17 **b)** 18 **c)** 45 **d)** 80 **e)** 81

7. A tiny factory produces apple smoothies, each of which requires two oranges and three apples, and orange smoothies, each of which requires two apples and four oranges. The factory has a supply of 51 oranges and 43 apples, and these are to be used so that as few fruits as possible will remain. What will be left unused?

a) nothing
b) at least one orange
c) at least three oranges
d) an orange and an apple
e) three apples and three oranges

8. If we agree that $a \star b$ means the same as $ab + 1$, then what is

$$(1 \star 2) \star (3 \star 4) \ ?$$

**a)** 25 **b)** 27 **c)** 35 **d)** 37 **e)** 40

9. Given a triangle $\triangle ABC$, how many points P exist, for which the points A, B, C and P are (in some order) the vertices of some parallelogram?

a) 1 **b)** 2 **c)** 3 **d)** 4 **e)** 5

10. A mongoose shakes a tree so that coconuts would fall down. It has to shake the tree for five minutes to drop one nut. When the first coconut hits the ground, a sloth wakes up and goes to rob nuts. The sloth needs seven minutes to crawl from its nest to the tree, and it needs the same time to get back to its nest with its spoil. The sloth can only carry one nut at a time, and it continues stealing nuts until the mongoose has left with its catch. The mongoose wants 15 nuts. How much time does it take to achieve that goal?

a) 75 min **b)** 115 min **c)** 145 min **d)** 150 min **e)** 250 min

11. The midpoints of the sides of a regular hexagon are joined and in this way a new hexagon is formed. What is the area of the smaller hexagon, if the area of the larger hexagon is 1?
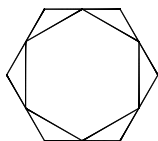
a) $\frac{1}{2}$ **b)** $\frac{2}{3}$ **c)** $\frac{3}{4}$ **d)** $\frac{4}{5}$ **e)** $\frac{5}{6}$

12. The numbers

$$0^2, \quad 1^2, \quad 2^2, \quad 3^2, \quad 4^2, \quad \dots, \qquad \text{i.e.} \qquad 0, \quad 1, \quad 4, \quad 9, \quad 16, \quad \dots,$$

are called square numbers. When a square number is divided by seven, what are the possible remainders?

a) 0 and 1 **b)** 0, 1 and 2 **c)** 0, 1 and 4 **d)** 0, 1, 2 and 4 **e)** 0, 1, 2, 3, 4, 5 and 6

13. How many quadrilaterals are there in which the sum of any three angles is smaller than 270°?

a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** at least four