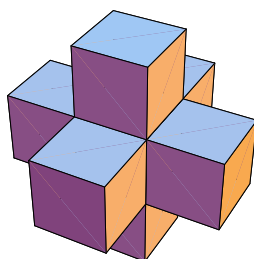


FINAL ROUND 15 MARCH 2014

1. In a desert, there is a 1000 kilometres long circular road which goes through five cities. When driven to the clockwise direction, the distance from the city A to the city B is 350 km, the distance from B to the city C is 120 km, from there to the city D 90 km, and from D to E 180 km, and finally from E back to A the circle is closed by a road 260 km long. In each city, there is a van which consumes 10 litres of gasoline per hundred kilometres, but its tank is empty. There is one gasoline canister in each city. In the city A there is a canister with 41 liters of gasoline, in B there are 8 litres, in C the canister contains 6 liters, in D 22 litres and in E 23 litres. Thus, the total amount of gasoline available is just the required 100 liters. A helicopter brings you to any of the cities and you get to choose which. Then your job is to drive through the entire road in the clockwise direction and finally return to the city where you began your 1000 km journey. Which city should you choose? The van can carry all the canisters you find and therefore they can be used later.

2. The following picture depicts an object which is formed when we take a steel cube (with a one metre long edge) and weld a similar cube to each of its faces.



The object is placed into a cubical pool (which is open from the top). An edge of the pool is 3 metres long, and therefore this object fits into the pool just so and so. In the beginning, there is water in the pool, and the depth of the water is 2 metres. How much does the level of the water rise when the object is placed to the pool so that it touches the floor of the pool?

3. Find all positive integers x and y for which

$$x^3 + y^2 = 100.$$

4. Is it possible to place nine different positive integers from the interval $[1, 15]$ into a 3×3 -grid so that the sums of the (horizontal) rows are also different positive integers from the interval $[1, 15]$?

5. A lord is murdered in his study. The upper floor of the manor is depicted below. It has 15 rooms in a 3×5 -grid. The study of the lord is at the corner and the room in the middle is a living room with a fireplace. The ghost of the lord does not know who the murderer was, but he knows that the murderer lives in one of the 13 other rooms of the upper floor (that is, excluding the study and the living room). Starting from the study, the ghost wants to visit and haunt each of the rooms, exactly once each, and in the end leave through the chimney of the living room. The ghost is able to pass through the walls but not through the corners. The picture below depicts one possible path of the ghost. However, please note that this path does not go through all the rooms and so does not satisfy all the wishes of the ghost. Help the ghost find a suitable path through the rooms, or explain why such a path does not exist and the ghost is doomed to haunt the manor forever.

