Mathematics Competition
for the Seventh Graders of Oulu Sub-Region
Final Round 23.4.2016

1. Pascal's triangle is constructed in the following way. The first row consists of just the number 1. On the second row there are two 1's. The next row is always one number longer than the one above it. The row is formed by putting 1's on both ends of the row, a bit further out than on the previous row. The rest of the row is formed by writing the sum of every two adjacent numbers on the previous row directly below them. The triangle has an infinite number of rows.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1 \\
\end{array}
$$

$\cdots\cdots\cdots\cdots\cdots\cdots$

How many times does the number 10 occur in Pascal's triangle?

2. Every Tuesday Matti, Heta, Juha, Oona, Reetta and Teemu meet for a math club. The students gather around a round table with six chairs. Matti always sits in the same place. Heta wants to sit next to Matti. Oona and Reetta don't get along well, and thus they don't want to sit next to each other. How many possible ways are there for the students to sit around the table?

3. Taavetti buys a peculiar gadget from a flea market. It has three buttons, one yellow, one red and one blue, and a wheel. The salesman explains that each button corresponds to some number $1, 2, 3, 4, \cdots$. He remembers that the yellow button corresponds to the number 7. He can't say anything about the other two buttons. The gadget works in the following way. If you push a button once, the wheel turns a number of rounds equal to the value of the button. If you push a button quickly twice in a row, the wheel turns a number of rounds corresponding to the value of the button multiplied by itself. Thus, if you push the yellow button, the wheel turns 7 rounds and if you push the yellow button twice in a row, the wheel turns 49 times. Taavetti pushes the red button once and the blue button once. The wheel turns 5 rounds. Then he pushes the blue button twice in a row and then the red button once. The wheel turns 11 rounds.

What are the values of the red and blue buttons?

4. There are 25 light bulbs in a room, numbered from 1 to 25. Each light bulb has a switch. At the start, all light bulbs are switched off. First, we toggle all the switches whose number is divisible by one $(1, 2, 3, \ldots, 25)$. Then we toggle each divisible by two $(2, 4, 6, \ldots, 24)$, then by three and so on until finally we toggle each divisible by 25.

Which lights are on at the end and why?

5. Let's play a single player game called Game of Life on a grid paper. The 8 squares around a square closest to it are called its neighbors. At the start some squares on the grid contain a live cell. On each round some cells may stay alive, other cells may die and some new ones might be born on empty squares. This happens according to the following rules:

1. If a cell has exactly two or three neighboring cells, it stays alive.

2. If a cell has just one or no neighboring cells, it dies.

3. If a cell has more than three neighboring cells, it dies.

4. If an empty square has exactly three neighboring cells, a new cell is born on the square.

The births and deaths of cells according to the above rules happen simultaneously.

For example consider the following cell formation:
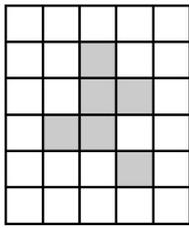
In one round the cell formation changes in the following way:
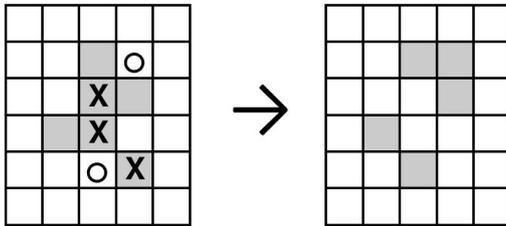
We denote live cells by grey squares. An X denotes the death of a cell, and a small circle denotes the birth of a new cell.

a) What does the following cell formation look like after two rounds? What about after a hundred rounds?
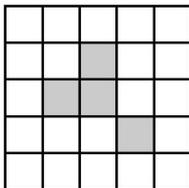
b) Find a cell formation which dies completely in exactly four rounds.