Mathematics Competition for the Seventh Graders of Turku Region, 5–9 March, 2018

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, paper, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $-3 \cdot 14$.

a) 0 **b)** -11 **c)** 11 **d)** -42 **e)** 42

2. Compute $2 \cdot (-|\frac{2}{|-4|}| + \frac{1}{2})$.

a) -2 **b)** -1 **c)** 0 **d)** 1 **e)** 2

3. Two cars both make a 120 kilometer trip. One of the cars is driven at 100 km/h whereas the other is driven at 80 km/h. The cars embark on the journey at the same time, but one of the cars stops for a break, whereas the other does not. How long was the break if both cars arrive at the destination at the same time?

a) 5 min **b)** 10 min **c)** 12 min **d)** 15 min **e)** 18 min

4. Let V_s be the volume of a rectangular $1 \times 2 \times 3$ parallelepiped, and let V_k be the volume of a $1 \times 1 \times 1$ cube. What is $\dfrac{V_s}{V_k}$?

a) 1 **b)** 6 **c)** 18 **d)** 36 **e)** 216

5. Ten students estimate the price of a liter of milk. Their estimates are

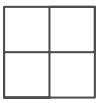
$$84, 85, 87, 90, 92, 94, 96, 99, 101 \text{ and } 103$$

cents. In the grocery story it turns out that at least half of the students estimated the price too high, that the price in cents is divisible by three and that two students gave estimates differing from the truth by exactly one cent. How many cents did a liter of milk cost?

a) 87 **b)** 91 **c)** 93 **d)** 96 **e)** 102

6. In how many ways can we color the following figure using black paint and white paint, if it is required that the figure has as much black paint as white paint, and that each small square is colored using one paint only?

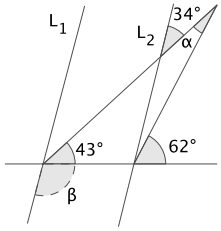
a) 1 **b)** 3 **c)** 4 **d)** 6 **e)** 8



7. Let x and y be fractions. What can we say (with certainty) about the number $x + y$?

a) It is an integer. **b)** It is at most 1. **c)** It is negative.
d) It is all of the previous. **e)** None of the previous can be said with certainty.

8. The lines L_1 and L_2 are parallel. Compute the angles α and β.



a) $\alpha = 19°$ and $\beta = 103°$ **b)** $\alpha = 34°$ and $\beta = 103°$
c) $\alpha = 19°$ and $\beta = 118°$ **d)** $\alpha = 34°$ and $\beta = 118°$
e) $\alpha = 34°$ and $\beta = 62°$

9. Let the sum of four consecutive integers be S. What is the remainder of S when it is divided by 4?

a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** The result depends on the four numbers in question.

10. For how many integers x do we have $2x^{2018} = 100000000001$? (The notation x^{2018} means the number $x \cdot x \cdot \ldots \cdot x$, where x appears 2018 times.)

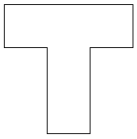
a) 0 **b)** 1 **c)** 2 **d)** 100 **e)** infinitely many

11. Let us consider the table below. What is the product of the numbers in the table? (The notation a^n means the product $a \cdot a \cdot \ldots \cdot a$, where a appears n times.)

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

a) $2^5 \cdot 3^5 \cdot 5^5$ **b)** $2^{10} \cdot 3^5 \cdot 5^5$ **c)** $2^{15} \cdot 3^5 \cdot 5^5$
d) $2^{10} \cdot 3^{15} \cdot 5^5$ **e)** $2^{30} \cdot 3^{10} \cdot 5^{10}$

12. Below is a letter T, and its width is 5 and its height is 7. All the angles are right angles. Determine the length of the boundary line.



a) 24 **b)** 20 **c)** 17 **d)** 28
e) Cannot be determined with the given data.

13. Let us form from the numbers -1, 0, 1 and 2 all possible pairs where the two numbers are different. Let us compute for each pair the product of the numbers in the pair. What is the proportion of pairs for which the product is equal to zero?

a) $\frac{1}{6}$ **b)** $\frac{1}{5}$ **c)** $\frac{1}{4}$ **d)** $\frac{1}{3}$ **e)** $\frac{1}{2}$

14. Let us define a new operation $\star$ by setting $a \star b = a + 2b$. If a is an arbitrary given number, does there always exist a number b such that $a \star b = 0$?

a) No, such a number exists only if $a = 0$. **b)** Yes, any b has that property.
c) No, such a number never exists.
d) Such a number exists only when $a = 0$ or $a = 1$. **e)** Yes, $b = -a/2$ is such a number.

15. In how many ways can we write the numbers 1, 2, ..., 9 in a row so that the sum of any two consecutive numbers is at least 10, and that the sum of the first and the last numbers is at least 11?

a) 0 **b)** 1 **c)** 5 **d)** 10 **e)** 100