Mathematics Competition for the Seventh Graders of Helsinki, 4–8 March, 2019

- The time allotted is 50 minutes.
- The allowed tools are writing and drawing instruments, i.e. pencil, eraser, ruler and compass. Calculators and mathematical tables are not allowed.
- Each problem is worth one point. Wrong answers are not punished.
- The problems are not ordered in increasing difficulty, but the first problems are likely to be easier than the last ones.

1. Compute $23 + 57 + 89$.

a) 147 **b)** 169 **c)** 172 **d)** 181 **e)** 195

2. Compute $24 \cdot 15 - 25 \cdot 18$.

a) -84 **b)** -86 **c)** -88 **d)** -90 **e)** -92

3. A flight from Helsinki to Beijing takes 7 hours and 35 minutes. The local time in Beijing is five hours ahead of the local time in Helsinki. If the flight takes off from Helsinki at 18:20, then what is the local time in Beijing when the flight lands there?

a) 05:45 **b)** 06:55 **c)** 15:45 **d)** 18:55 **e)** 20:55

4. A deciliter of flour weighs about $65\,\mathrm{g}$, and a deciliter of oatmeal weighs about $35\,\mathrm{g}$. A recipe for an apple pie requires, in deciliters, half as much oatmeal as flour. If $520\,\mathrm{g}$ of flour is used, how much oatmeal should be used?

a) $100\,\mathrm{g}$ **b)** $140\,\mathrm{g}$ **c)** $220\,\mathrm{g}$ **d)** $740\,\mathrm{g}$ **e)** $320\,\mathrm{g}$

5. A swimming pool has the shape of a rectangular parallelepiped, its depth is 2.5 meters, its length is 25 meters and its width is 10 meters. It is filled with two meters of water at the pumping rate of 100 liters per minute. How many minutes does the operation take?

a) 5 **b)** 250 **c)** 500 **d)** 2500 **e)** 5000

6. All the angles marked in the star diagram are equal to α, and all the sides in the diagram are equally long. How large is the angle α?

a) 34° **b)** 35° **c)** 30° **d)** 45° **e)** 36°
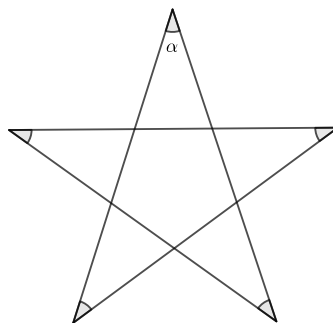
7. What is the last digit of the number $1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot 17 \cdot 19$?

a) 0 **b)** 1 **c)** 2 **d)** 3 **e)** 5

8. Compute the sum of all the numbers in the table.

| 10	20	30	40	50
20	40	60	80	100
30	60	90	120	150
40	80	120	160	200
50	100	150	200	250

a) 500   **b)** 1000   **c)** 2250   **d)** 3560   **e)** 4550

9. The average of three numbers is 10 and the average of two other numbers is 5. What is the average of all five numbers?

a) 3   **b)** 5   **c)** 6.5   **d)** 7.5   **e)** 8

10. The area of a rectangle is 1 and it has been divided into four parts by segments parallel to the sides. Compute the area of the black region.

a) $\dfrac{1}{4}$ **b)** $\dfrac{3}{8}$ **c)** $\dfrac{1}{3}$ **d)** $\dfrac{7}{16}$ **e)** $\dfrac{1}{2}$
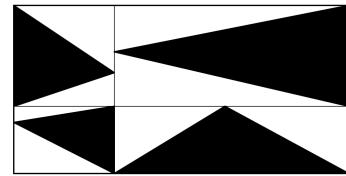
11. How many pairs of integers such that the sum of the integers is 2019 and the product of the integers is 2019 are there?

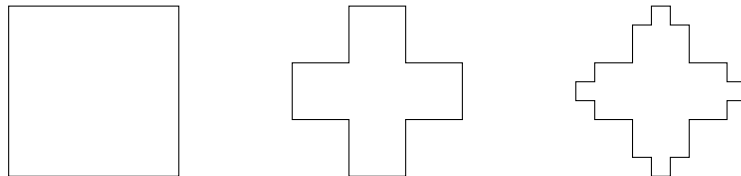**a)** 0   **b)** 1   **c)** 10   **d)** 100   **e)** more than 1000

12. Compute $\dfrac{2}{10} \cdot \dfrac{4}{9} \cdot \dfrac{6}{8} \cdot \dfrac{8}{7} \cdot \dfrac{10}{6} \cdot \dfrac{12}{5} \cdot \dfrac{14}{4} \cdot \dfrac{16}{3} \cdot \dfrac{18}{2} \cdot \dfrac{20}{1}$.

a) $\dfrac{142}{13}$ **b)** 512   **c)** 743   **d)** 1024   **e)** $\dfrac{32468}{7}$

13. In a certain year, the month of March has exactly four Mondays and four Fridays. What day of the week is the 31st day of March?

a) Monday   **b)** Tuesday   **c)** Wednesday   **d)** Thursday   **e)** Friday

14. A sheet of paper has the shape of a square. In the first round, small squares are cut from the corners, so that the resulting polygon has 12 corners as in the diagram below. In the second round, small squares are again cut from the 90° corners so that the resulting polygon has 28 corners. In each round, we keep repeating this operation of cutting small squares from the 90° corners. How many corners (both 90° and 270° corners) the polygon has after the fifth round of cutting?

a) 68   **b)** 100   **c)** 124   **d)** 128   **e)** 252

15. In how many ways can one choose positive integers x, y, z and w so that

$$x^2 + y^2 = 2(z^2 + w^2)?$$

a) 1   **b)** 18   **c)** 63   **d)** 100   **e)** more than 100