Mathematics Competition for the Seventh graders of Oulu Sub-Region, 18–22 February 2019
Solutions

1. Compute $-9 \cdot 7 + 198$.

a) -251 **b)** 135 **c)** 53 **d)** 251 **e)** 33

Solution. By a direct calculation we get $-9 \cdot 7 + 198 = -63 + 198 = 135$.

2. Compute $1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}$.

a) $\frac{1}{10}$ **b)** $\frac{4}{3}$ **c)** 1.234 **d)** $\frac{43}{30}$ **e)** $\frac{25}{12}$

Solution. Let's calculate:

$$1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}} = 1 + \cfrac{1}{2 + \cfrac{4}{13}} = 1 + \frac{13}{30} = \frac{43}{30}.$$

3. There is a discount offer in a shop: you get six bags of candies for the price of five. One bag costs 3 euros. How many bags of candies can you get with 50 euros?

a) 16 **b)** 17 **c)** 18 **d)** 19 **e)** 20

Solution. Since $50/3 = 16{,}66666\ldots$, normally you'd get 16 bags with 50 euros. Now you get one bag more for every five bags, which is 3 bags more in this case. The correct answer is thus 19.

4. Compute $-1 + 2 - 3 + 4 - 5 + 6 - \ldots - 2017 + 2018 - 2019$.

a) -3028 **b)** 0 **c)** 2020 **d)** -1009 **e)** -1010

Solution. We can think of the numbers as consecutive pairs until the number 2018. There are 1009 of these pairs, each of which produces the number 1. Finally we subtract the number 2019, so the result is

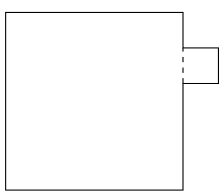
$$1009 - 2019 = -1010.$$

5. The leader of a summer camp is informed of the number of participants attending the camp. She immediately notices that there must be at least three participants born in the same month. How many participants are there at least?

a) 3 **b)** 12 **c)** 24 **d)** 25 **e)** 36

Solution. There are 12 months in a year. Because at least three participants have their birthdays in the same month, there must be at least $2 \cdot 12 + 1 = 25$ participants in the camp.

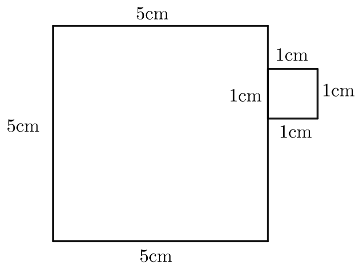
6. In the picture below, the side length of the bigger square is 5 cm and the side length of the smaller square is 1 cm. Calculate the perimeter of this object.



a) 5 cm **b)** 12 cm **c)** 22 cm **d)** 24 cm
e) The problem can not be solved with the given information.

Solution. Because the side length of the bigger square is 5 cm and the side length of the smaller square is 1 cm, we get side lengths as indicated in the picture below. Hence the perimeter of the object is

$$3 \cdot 5\text{cm} + 3 \cdot 1\text{cm} + (5\text{cm} - 1\text{cm}) = 22\text{cm}.$$



7. The price of a digital camera in an electronics store is 100 euros at the beginning. The price of the camera first drops 20 % and then rises 20 %. What is the price of the camera after these changes?

a) 24 € **b)** 96 € **c)** 100 € **d)** 104 € **e)** 120 €

Solution. The new price is $100 \cdot 0.8 \cdot 1.2 = 96$ euros.

8. A square is inscribed inside a circle of radius one (i.e., its vertices are located on the boundary of the circle). What is the area of this square?

a) 1 **b)** 2 **c)** π **d)** 3 **e)** $\frac{\pi}{2}$

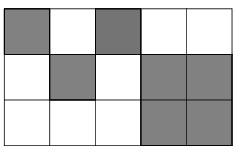
Solution. Because the vertices of the square are on the boundary of the circle, the distance from a vertex to the centre is 1, implying that the diameter is 2. Hence the side of the square is $\sqrt{2}$ and area 2.

9. Ulla, Leena, and Ville go for a hiking trip. They drive a car to the starting point of the hiking trail and start walking along the trail with the speed of 6 km/h. After walking ten minutes, they notice that the sausages were left in the car, and Ulla starts jogging along the trail back to the car with the speed of 12 km/h. Meanwhile, Leena and Ville continue walking along the trail with the speed of 6 km/h. How far from the car have Leena and Ville got when Ulla gets to the car?

a) 1 km **b)** 2 km **c)** 1.5 km **d)** 0.5 km **e)** 2.5 km

Solution. During the first ten minutes, the three of them have walked 1 kilometre. Ulla jogs this distance back and at the same time Leena and Ville walk along with half the speed of Ulla, so they proceed another 0.5 km. The answer is thus 1.5 km.

10. A rectangle has been divided into 15 congruent squares, as indicated in the picture. The area of the shaded region is 14. What is the area of the whole rectangle?



a) 15 **b)** 24 **c)** 30 **d)** 35 **e)** 42

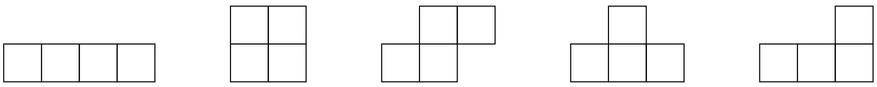
Solution. The area of one small square is $\frac{1}{7} \cdot 14 = 2$. Hence the area of the whole rectangle is $15 \cdot 2 = 30$.

11. The sum of the numbers a and b is 42 and their difference is 20. What is the product of the numbers a and b?

a) 143 **b)** 210 **c)** 341 **d)** 840 **e)** 1364

Solution. This problem can be solved in several ways. One possible way is to simply solve $a = 31$ and $b = 11$, whence the product is $11 \cdot 31 = 341$.

12. Squares of the same size are used to form patterns by attaching them side by side so that every square is attached to another from at least one side. Patterns are interpreted to be the same if they can be formed from each other by rotation or reflection. With four squares one can form the following 5 different patterns:



How many different patterns can be formed using five squares?

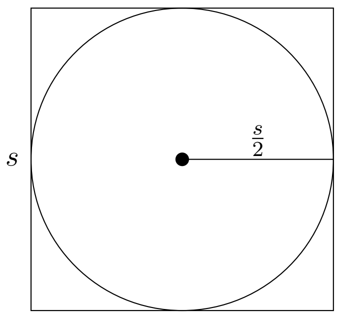
a) 10 **b)** 12 **c)** 14 **d)** 16 **e)** 18

Solution. All the squares can be placed in the same row, so we get one pattern. If they are placed in two rows, the distribution can be 2 and 3 or 1 and 4. If the distribution is 2 and 3, there are three options, and if it is 1 and 4, there are four options. If the squares are placed in three rows, one row has to have three squares and the other two rows one square each. Then there are four options. Altogether we get $1 + 3 + 4 + 4 = 12$ different patterns.

13. Let the side length of a square be s and the radius of a circle be $\frac{s}{2}$. What can be said about the area N of the square and the area Y of the circle?

a) $N < Y$ **b)** $N = Y$ **c)** $N > Y$ **d)** All of the former **e)** None of the former

Solution. Consider a square with side length s. One can inscribe a circle of radius $\frac{s}{2}$ inside the square, so that the two objects have the same centre, as indicated in the picture. Because the circle covers only a part of the area of the square, we have $N > Y$.

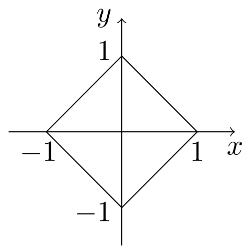


14. Consider the product of n consecutive positive integers, where $n \geq 2$ is a positive integer. Which of the following numbers certainly divides this product?

a) 3 **b)** 10 **c)** n **d)** All of the former **e)** None of the former

Solution. First consider the case $n = 2$. Then the consecutive integers can be, for example, 1 and 2. Since $1 \cdot 2 = 2$ and 2 is not divisible by the numbers 3 or 10, the product under consideration is not necessarily divisible by 3 or 10. On the other hand, exactly one of n consecutive positive integers is divisible by n. Hence also their product is divisible by n. Therefore **c** is the correct answer.

15. Which of the following equations corresponds to the quadrangle below?



a) $y + x = 1$ **b)** $|y| - |x| = -1$ **c)** $|y| + |x| = -1$ **d)** $|y| + |x| = 1$ **e)** $|y| - |x| = 1$

Solution. The picture corresponds to the situation where the sum of the absolute values of the x and y coordinates is 1.