Mathematics Competition for the Seventh graders of Oulu Sub-Region, Final 17.4.2021

Remember to justify every step of your solution!

1. Jussi has two ten litre buckets and one three litre bucket. There is nine litres of water in one of the big buckets and the other is exactly half full. The three litre bucket is empty.

Jussi wants to measure exactly seven litres of water in each of the big buckets and keep the small bucket empty. How can Jussi do this? He can pour water from a bucket to another but, upon doing so, has to keep on pouring until either one of the buckets is full or empty. Jussi can't get more water anywhere and he has no other tools available.

2. You find a machine that has five levers in a row. The lengths of the levers from left to right are $3, 2, 1, 2$ and 3. By pulling all of the levers in correct order you open a hidden door and find a treasure. On your previous adventures you have collected the following clues to help you determine the correct order for pulling the levers:

1) The first lever has no longer lever next to it.
2) The second lever is on the right side of the shortest lever.
3) The third lever is shorter than any of the levers pulled so far.
4) The fourth lever is not the shortest nor the longest lever.
5) The last lever has a lever on both sides of it and one of them is the lever that was pulled second.

What is the correct order for pulling the levers to open the hidden door?

3.

a) Determine and justify whether the following claim is true or false: Whenever two polygons have the same perimeter then they also have the same area.

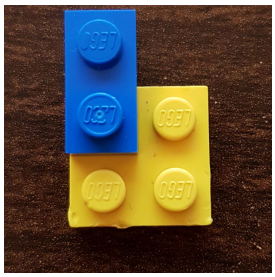
b) Give an example of such a rectangle that the numeric value of its perimeter equals the numeric value of its area.

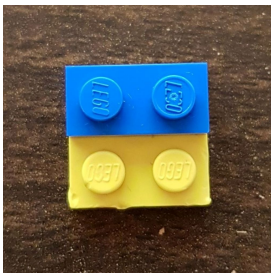
4. Determine all integers x and y that satisfy the equation

$$xy + x = 23.$$

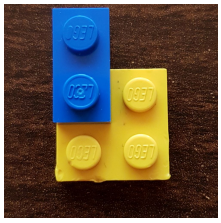
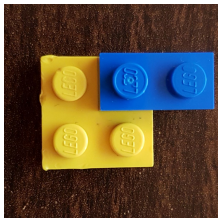
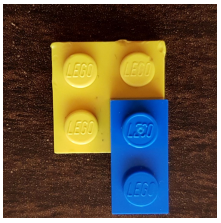
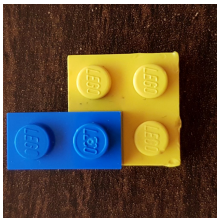
Task 5 is on the next page.

5. The blue 1×2 Lego brick can be attached on top of the yellow 2×2 Lego brick in three different ways:





When counting the number of different ways to attach the bricks, the cases where one way can be attained by rotating another way are counted as one. For example all four ways shown below are considered as same when counting the ways to attach the bricks:

How many different ways are there to attach the blue 1×2 Lego brick on top of the yellow 4×4 Lego brick when the ways attained by rotation are considered the same?