Mathematics Competition for the Seventh graders of Oulu Sub-Region, 22–26 February 2021
Solutions

1. Compute $1-2+3-4+5$.

a) -1 **b)** 0 **c)** 1 **d)** 2 **e)** 3

Solution. e) 3:

$$1-2+3-4+5=1+(-2+3)+(-4+5)=1+1+1=3$$

2. Compute $\frac{2\cdot4\cdot6\cdot8\cdot10}{1\cdot2\cdot3\cdot4\cdot5}$.

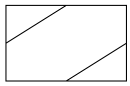
a) $\frac{1}{2}$ **b)** 2 **c)** 32 **d)** 120 **e)** 3840

Solution. c) 32:

$$\frac{2\cdot4\cdot6\cdot8\cdot10}{1\cdot2\cdot3\cdot4\cdot5}=\frac{2^5\cdot1\cdot2\cdot3\cdot4\cdot5}{1\cdot2\cdot3\cdot4\cdot5}=2^5=32$$

3. How many ways you can color the flag below when you have three colors and adjacent regions must be of different color? You don't have to use all the colours in every coloring.

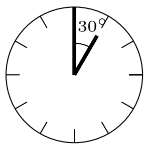
a) 3 **b)** 6 **c)** 9 **d)** 12 **e)** 15



Solution. d) 12: There are three options for the color of the region in the middle. After that choice there are two color options for each of the corners. Hence there are $3\cdot2\cdot2=12$ different colorings.

4. When the time is one o'clock the angle between the hands of the clock is 30°. What is the angle between the hands when the time is half past three?

a) 30° **b)** 45° **c)** 60° **d)** 75° **e)** 90°



Solution. d) 75°: When the time is half past three, the longer hand points directly downwards at six o'clock and the shorter hand points exactly at the middle of three o'clock and four o'clock. Since the angle between two numbers in the face of the clock is $360°/12=30°$, the shorter hand points 15° clockwise from three o'clock. Since the angle between three o'clock and six o'clock is 90°, the angle between the hands is $90°-15°=75°$ when the clock is half past three.

5. Sofia went shopping in three different stores. In the first store she used one third of her money and in the second store she used half of the money she had left. On the way to the third store she found a 10 euro bill which she took for herself. In the third store she used one fourth of the money she had at the moment. After the shopping Sofia had 18 euros left. How much money did Sofia have before she went shopping?

a) 12 **b)** 42 **c)** 72 **d)** 144 **e)** 372

Solution. b) 42: Let m denote the amount of money Sofia had before she went shopping. After visiting the first store she had $m-\frac{1}{3}m=\frac{2}{3}m$ euros left. In the second store she spent half of what se had so after shopping in the second store she had $\frac{\frac{2}{3}m}{2}=\frac{1}{3}m$ euros left. After finding the 10 euro bill she had $\frac{1}{3}m+10$ euros. After visiting the last store she had $\frac{3}{4}\left(\frac{1}{3}m+10\right)=18$ euros. Solving the equation gives $m=42$.

6. What is the last digit of number

$$1+2+3+4+\ldots+2019+2020+2021?$$

a) 1 **b)** 5 **c)** 7 **d)** 8 **e)** 0

Solution. a) 1: Now

$$1+2+3+4+\ldots+2019+2020+2021 = 2021 \cdot \frac{1+2021}{2} = 2021 \cdot 1011.$$

Since the last number of each of the factors is 1, so is also the last number of $2021 \cdot 1011$.

7. How many positive two-digit numbers there are so that the product of the digits is greater than the number itself? For example number 29 is not such a number since $2 \cdot 9 = 18 < 29$.

a) 1 **b)** 3 **c)** 5 **d)** 7 **e)** None

Solution. e): A positive two-digit number is of the form $10a+b$, where a and b are integers between 0 and 9. Since $b \leq 9$, then $ab \leq 9a < 10a+b$.

8. We know that $\triangledown$ is a number operation. We also know that $5 \triangledown 3 = 3 \triangledown 5$ and number $\frac{4 \triangledown 4}{2 \triangledown 4}$ is an integer. Which number operation can $\triangledown$ be?

a) Addition **b)** Subtraction **c)** Multiplication **d)** Division **e)** None of those

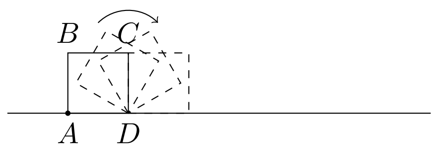
Solution. c): By the first condition operation $\triangledown$ cannot be subtraction or division because $5-3=2 \neq -2 = 3-5$ and $\frac{5}{3} \neq \frac{3}{5}$. By the second condition the operation cannot be addition since $\frac{4+4}{2+4} = \frac{8}{6}$ is not an integer. Multiplication, however, satisfies both conditions because $5 \cdot 3 = 15 = 3 \cdot 5$ and $\frac{4 \cdot 4}{2 \cdot 4} = 2$.

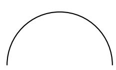
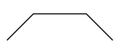
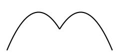
9. Matti wants to find out how many matchsticks there are left in his matchbox. He knows that there used to be 70 matches in the box. Matti discovers that by using **all** of the matches left in the box he can form an equilateral triangle, a square or a regular pentagon. How many matchsticks are there left in the matchbox?

a) 12 **b)** 25 **c)** 40 **d)** 60 **e)** 70

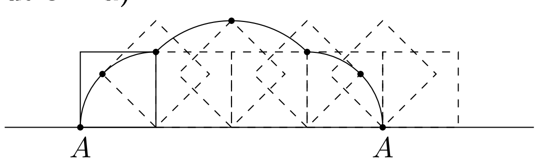
Solution. d) There must be an equal amount of matchsticks in each side of a regular polygon. Therefore the number of matchsticks must be divisible by 3, 4 and 5. The smallest such number is $3 \cdot 4 \cdot 5 = 60$. All the other numbers divisible by 3, 4 and 5 are greater than 70.

10. Square $ABCD$ is rolled on a flat surface always around the lower right corner of the square. The square is rolled until vertex A is the lower left corner of the square again. Which pattern does vertex A draw on the plane during the rolling?



a)  **b)**  **c)**  **d)**  **e)** 

Solution. a):



11. Jarmo has 1,5 kg of green yarn, 2 kg of white yarn and 3 kg of black yarn. He plans to make as many woolen socks of the yarn as possible. One woolen sock requires 35 g of green yarn, 55 g of white yarn and 70 g of black yarn. Which yarn he runs out of first?

a) Green. **b)** White. **c)** Black. **d)** Green and black at the same time. **e)** White and black at the same time.

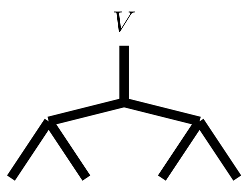
Solution. b): Since

$$\frac{1{,}5 \text{ kg}}{35 \text{ g}} = \frac{1500 \text{ g}}{35 \text{ g}} \approx 42{,}9$$

there is enough of green yarn for 42 woolen socks. Similarily $2000/55 \approx 36{,}4$ and $3000/70 \approx 42{,}9$ so there is enough of white yarn for 36 woolen socks and black yarn for 42 woolen socks. Therefore Jarmo runs out of white yarn first.

12. Water is poured into the pipeline below through the input V and the water comes out from the four outputs at the bottom of the pipeline. The pipeline is built of blue and red pipes. Water flows through a blue pipe in 1 second and through a red pipe in 2 seconds. How many ways there are to build the pipeline so that the water comes out from all four outputs at the same time?

a) 11 **b)** 12 **c)** 13 **d)** 14 **e)** 15



Solution. b) 12: Water comes out from all outputs at the same time only if each possible route has exactly the same number of blue and red pipes. We get two cases when all the pipes are of same color. If each route has exactly one blue pipe, there are 5 different possibilities: If the topmost pipe is blue, all the others are red. If the topmost pipe is red then each color combination of the two pipes in the middle corresponds to one case (if a middle pipe is blue then the following pipes at the bottom are red, and if a middle pipe is red then the following pipes at the bottom are blue). Similarily if each route has exactly one red pipe, there are 5 different cases. Combined there are 12 different possibilities.

13. Integers $1, 2, \ldots, 10$ are colored by the following rule: If integer a is colored with color V, then none of integers $a+1, a+2, \ldots, a+a$ is colored with color V. How many colors at least do you need to color integers $1, 2, \ldots, 10$?

a) 3 **b)** 4 **c)** 5 **d)** 6 **e)** 10

Solution. d): If number 1 is colored with color V_1 we can color numbers 3 and 7 with V_1 as well. For numbers 2 and 5 we can use a different color V_2. For numbers 4 and 9 we need a new color again, say V_3. The rest of the numbers, that is $6, 8$ and 10, must all be of different colors so we need three more colors V_4, V_5 and V_6.

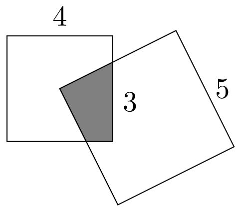
14. Which of the following claims is true?

a) If an integer is divisible by three then it is always odd.
b) If an integer is odd then it is always divisible by three.
c) If an integer is divisible by three then it is always even.
d) If an integer is even then it is always divisible by three.
e) None of the claims above is true.

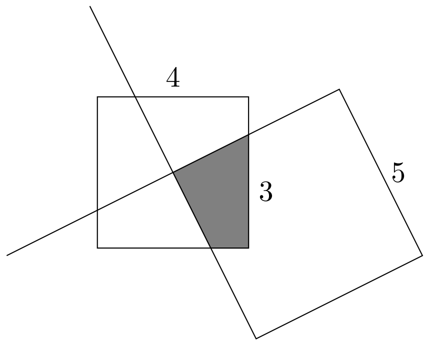
Solution. e) : Claim a) is false because for example number 6 is divisible by three but it is not odd. Claim b) is false because for example number 7 is odd but it is not divisible by three. Claim c) is false because for example number 3 is divisible by three but it is not even. Claim d) is false because for example number 2 is even but it is not divisible by three. Therefore the correct answer is e).

15. One vertex of a square with side-length 5 is at the midpoint of a square with side-length 4. What is the area of the colored region when the length of its vertical side is 3?

a) 3 **b)** 4 **c)** 5 **d)** 6 **e)** 7



Solution. b) 4: By drawing two lines through two sides of the larger square we can divide the smaller square into four congruent regions, one of which is the colored region.



Hence the area of the colored region is one quarter of the area of the smaller square, that is $4 \cdot 4/4 = 4$.