Mathematics Competition for the Seventh graders of Oulu Sub-Region, 28 February – 4 March 2022
Solutions

1. Compute $1 + 22 + 333 + 4444 + 55555$.

a) 59245 **b)** 60355 **c)** 65432 **d)** 65555 **e)** 666666

Solution. **b)** 60355:

$$1 + 22 + 333 + 4444 + 55555 = 1 + 355 + 59999 = 60000 + 355 = 60355$$

2. Compute

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6}.$$

a) $\frac{1}{11}$ **b)** $\frac{3}{11}$ **c)** $\frac{1}{5}$ **d)** $\frac{5}{6}$ **e)** 1

Solution. **e)** 1:

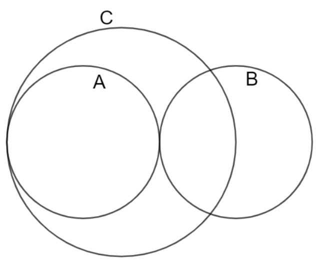
$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{12} + \frac{4}{12} + \frac{2}{12} = \frac{6+4+2}{12} = \frac{12}{12} = 1.$$

3. Huey, Dewey and Louie have chosen together one number between 1-20. Each one of them tells one clue about the number, but one of them is lying. Dewey says, that the number is divisible by seven. Huey says, that the number is divisible by five. Louie says, that the number is divisible by three. Which number did they choose?

a) 14 **b)** 15 **c)** 20 **d)** 16 **e)** 9

Solution. **b)** 15. Only the statements of Huey and Louie can both be true at the same time, otherwise the number they chose would be either at least $3 \cdot 7 = 21 > 20$ or at least $5 \cdot 7 = 35 > 20$. Therefore the number they chose must be divisible by 3 and 5, and the only such number between 1 and 20 is 15.

4. Circles A and B touch each other and they both have radius of 1. Circle C touches circle A as pictured and passes through the center of circle B. Furthermore the touching point of A and C is on the same line as the center points of circles A and B. What is the radius of circle C?



a) 1 **b)** $\frac{4}{3}$ **c)** $\frac{3}{2}$ **d)** 2 **e)** 3

Solution. **c)** $\frac{3}{2}$: The diameter of circle C is the sum of the diameter of circle A and the radius of circle B, that is $2 \cdot 1 + 1 = 3$. Hence the radius of circle C is $\frac{3}{2}$.

5. How many different ways can number 2022 be written as the sum of two positive integers if the order of the terms doesn't matter? Then, for example, sums $1 + 2021$ and $2021 + 1$ are considered as the same way.

a) 1010 **b)** 1011 **c)** 2020 **d)** 2021 **e)** 2022

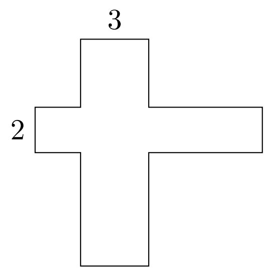
Solution. **b)** 1011: Since the order of the terms doesn't matter, we may assume that the first term is at most the second term. Since the smaller of the terms is at most half of 2022, that is 1011, then the first term is one of numbers $1, 2, \ldots 1011$. Hence there are 1011 different sums that give 2022.

6. Essi and Ossi are 150 meters apart from each other. They start walking towards each other at the same pace. The length of Essi's stride is 70 cm and the length of Ossi's stride is 80 cm. How many steps has Essi taken when they meet?

a) 94 **b)** 100 **c)** 107 **d)** 188 **e)** 214

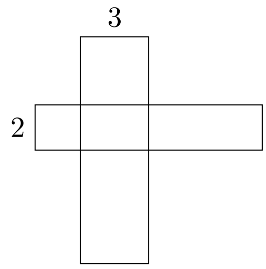
Solution. **b)** 100: Essi and Ossi meet, when they have walked 150 meters combined. Since they walk at the same pace, they have both taken k steps when they meet. Then $70k + 80k = 15000$ so $k = 100$.

7. What is the area of the figure below, when the height of the figure is 10 units and the width of the figure is 10 units?



a) 44 **b)** 48 **c)** 50 **d)** 56 **e)** 60

Solution. **a)** 44: The figure can be interpreted as two rectangles that cross each other



The are of the tall rectangle is $3 \cdot 10 = 30$ and the area of the wide rectangle is $2 \cdot 10 = 20$. The area of the whole figure is the sum of the areas of the two rectangles minus the area of the region where the two rectangles overlap. The area of the overlap region is $2 \cdot 3 = 6$ so the area of the figure is $30 + 20 - 6 = 44$.

8. Matti wants to read a book that is 1000 pages long. On the first day he reads one page, and after that, he reads as many pages every day as on the previous days combined. How long does it take for Matti to finish the book?

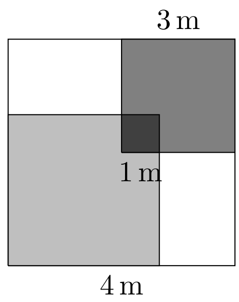
a) 7 days **b)** 11 days **c)** 20 days **d)** 35 days **e)** 114 days

Solution. **b)** 11 days. Matti has read $1 + 1 = 2$ pages on the second day, $2 + 2 = 4$ pages on the third day, $4 + 4 = 8$ on the fourth, $8 + 8 = 16$ on the fifth, $16 + 16 = 32$ on the sixth, $32 + 32 = 64$ on the seventh, $64 + 64 = 128$ on the eighth, $128 + 128 = 256$ on the ninth, $256 + 256 = 512$ on the tenth, and finally, $512 + 512 = 1024 > 1000$ pages on the eleventh day.

9. The length of the wall of a square-shaped room is 6 meters. Three square-shaped carpets are laid on the floor. The lengths of the carpets' sides are 3, 4 and 5 meters. What is the biggest area of the part of the floor that is definitely covered by **every** carpet, no matter how they are placed?

a) 0,5 m^2 **b)** 1 m^2 **c)** 1,5 m^2 **d)** 2 m^2 **e)** 3 m^2

Solution. **b)** 1 m^2. When the carpets of side lengths 3 and 4 meters are laid on the floor, the area covered by both of them cannot be less than 1 m^2. Indeed, the placement that leads to the smallest possible area covered by both of them is the one where the carpets are at the opposite corners of the room, as depicted in the picture below:



In the above placement, moving either of the carpets can only increase the area of the dark gray rectangle that is covered by both carpets. When the carpet of side length 5 meters is now laid on the floor, it will also surely cover this rectangle, no matter where it is placed.

10. One mathematician has a peculiar tree in their garden: On a rainy day the tree grows 10 cm, on a sunny day it grows 5 cm and on a cloudy day it grows 1 cm. After a 10-day holiday the mathematician measures that the tree has grown 58 cm. What kind of days were most of the ten days?

a) Rainy days. **b)** Sunny days. **c)** Cloudy days. **d)** There were an equal amount of each kind of days. **e)** The problem can't be solved with the given information.

Solution. **a)** Rainy days: On rainy and sunny days the tree grows an amount divisible by 5. When 58 is divided by 5 the remainder is 3 so the number of cloudy days must be 3 or 8. If there were 8 cloudy days the tree couldn't reach the growth of 58 cm in the remaining two days, so the number of cloudy days must be 3. Hence the tree grows $58 - 3 = 55$ cm in 7 rainy or sunny days. It is now quick to check that the only possibility is that there were 4 rainy days, 3 sunny days and 3 cloudy days: $4 \cdot 10 + 3 \cdot 5 + 3 \cdot 1 = 58$.

11. We know that

$$a + b + c + d + e + f + g + h + i + j = 101$$

and

$$a - b + c - d + e - f + g - h + i - j = 39.$$

What is $a + c + e + g + i$?

a) 31 **b)** 62 **c)** 70 **d)** 140 **e)** The problem can't be solved with the given information.

Solution. **c)** 70: By summing the equations we get

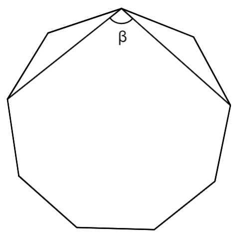
$$\begin{aligned} 2a + 2c + 2e + 2g + 2i &= 140 \\ 2(a + c + e + g + i) &= 140 \\ a + c + e + g + i &= \frac{140}{2} = 70. \end{aligned}$$

12. Certain regular polygon M can be divided by a line segment into two polygons, that have either 7, 8 or 9 interior angles combined. How many interior angles does polygon M have?

a) 3 **b)** 4 **c)** 5 **d)** 6 **e)** 7

Solution. c) 5. The two polygons can together have either two, three or four more angles than the polygon M: If the line drawn inside M connects two corners, then the number of angles increases by two. If the line connects a corner with an edge, the number of angles increases by three. If the line connects two edges, the number of angles increases by 4. Since $7 = 5 + 2$, $8 = 5 + 3$ and $9 = 5 + 4$, M must have 5 corners.

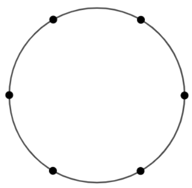
13. Two diagonals are drawn into a regular 9-gon as pictured below. What is the angle β between the two diagonals?



a) 40° **b)** 70° **c)** 90° **d)** 100° **e)** 140°

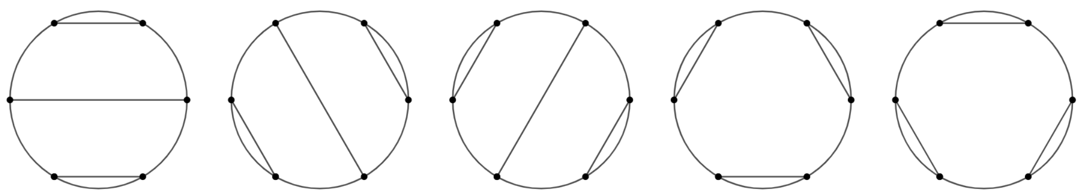
Solution. d) 100°: The interior angle of a regular 9-gon is $\frac{7\cdot 180°}{9} = 140°$. The diagonals and the perimeter of the 9-gon form two congruent isosceles triangles that have base angles of size $\frac{180°-140°}{2} = 20°$. Hence $\beta = 140° - 2 \cdot 20° = 100°$.

14. There are 6 points on the perimeter of a circle. How many different ways are there of combining the points pairwise with line segments so that none of the line segments intersect each other?



a) 2 **b)** 3 **c)** 4 **d)** 5 **e)** 6

Solution. d) 5: All possible ways are depicted below.



15. A phone manufacturer knows from experience that 2% of the phones, that are sent from the factory, are faulty. All faulty phones are eventually returned to the manufacturer. If 25% of all the returned phones were faulty already when they left the factory, what percentage of those phones that left the factory flawless are returned?

a) 2% **b)** 98% **c)** 75% **d)** 6% **e)** 27%

Solution. d) 6% (with decimal values rounded off). Since 2% of all of the phones make 25%, or one quarter, of the returned phones, we know that in total 8% of all of the phones that leave the factory are returned. Since 2% of all of the phones are faulty when they leave the factory, 6% of all of the phones that are sent from the factory leave the factory flawless but are returned at some point. Therefore, if N denotes the total number of phones sent from the factory, the proportion of the phones which are eventually returned of those which leave the factory flawless is $\frac{0{,}06 \cdot N}{0{,}98 \cdot N} \approx 0{,}06122 \approx 0{,}06 = 6\%$.