Mathematics Competition for the Seventh Graders of Oulu Sub-Region, Final 26.4.2025

Remember to carefully justify each step of your solution for each question!

1. ISBN is a 13-digit identifier assigned to books. Each publication has its own unique code, which distinguishes it from other books. The dashes within the identifier are not minus signs; they are included solely to make reading the code easier.

A verification method for ISBN identifiers detects minor errors: First, the digits in odd positions are selected (the 1st digit, 3rd, 5th, and so on, up to the 13th digit). Then, the digits in even positions are picked and each of them is multiplied by three. When all these 13 numbers are then summed up, the result is always divisible by ten.

a) The ISBN identifier of a book is $978-0-12-002151-2$. Verify with calculation, that the verification method explained above works for this book.

b) Another book has an ISBN identifier $978-0-01-0?1240-1$, but there is a smudge on the page that covers one of the digits. The smudge is denoted by a question mark. Find out what the hidden digit is.

2. Milka claims she is a mind-reader. She presents you with the following trick:

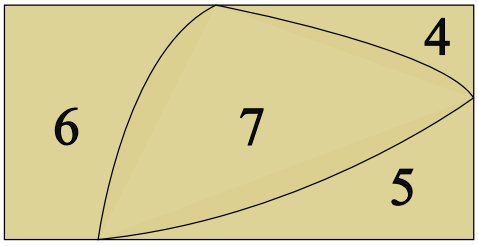
- "Think of a positive number, but don't reveal it to me," Milka says.
- She continues: "Now add one to that number."
- "Next, multiply the result by two."
- "Now add four to the number."
- "Next, divide the number you just got by two."
- "Finally, subtract the original number from the number you now have."
- "I know that the end result is three," Milka proclaims.

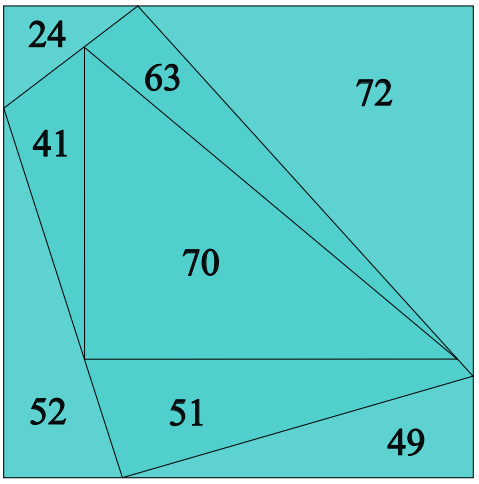
How did she know the end result?

a) Pick a number and show step by step, that the end result for the number you chose really is three.

b) Carefully justify, why the end result is always three regardless of the number you choose at the start. Milka is not a mind-reader after all.

3. The task is to determine the perimeter of a square/rectangle. The proportions of the images below are not accurate, so it is fruitless to try to take measurements from the images.



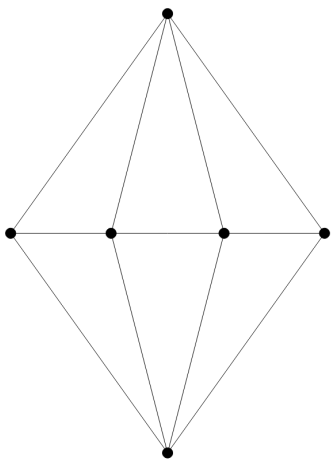


a) A rectangle is divided into four regions as shown in the image on the left. The perimeter of each region (i.e., the length of the boundary enclosing the region) is marked inside that region. What is the perimeter of the entire coloured rectangle?

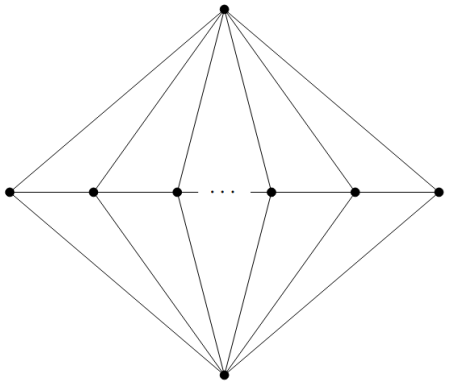
b) In the image on the right, a square is divided into eight regions. As in a), the number shown inside each region indicates the perimeter of that region. What is the perimeter of the entire colored square?

4. In this task you navigate along the lines of the graph shown in the image. The rules are as follows: you start at the topmost point, after which you are only allowed to move diagonally downwards as well as either to the left or right on the middle row. That is, if you have moved left or right on the middle row, you can no longer move in the opposite direction.

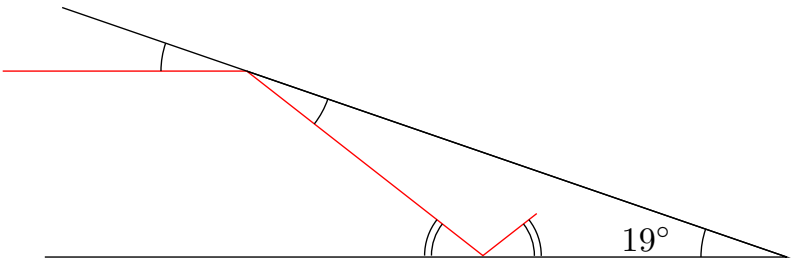
a) How many different paths are there from the topmost point to the point at the bottom?



b) Now, let's navigate in a graph where the middle row contains 2025 points. Note that the image below does not show all the points. How many different paths are there from the topmost point to the point at the bottom in this graph, while still following the same rules?



5. Two mirrors are placed at an angle of 19° relative to each other, as shown in the image below. A laser beam travels from left to right, parallel to the lower mirror, and reflects off the mirrors in such a way that, after each reflection, the angle between the laser beam and the mirror is the same as it was before the reflection. The situation depicted in the image below shows the laser beam after it has reflected twice. All the angles marked with a single arc in the image are equal to each other, as are both the angles marked with a double arc.



a) How many times does the laser beam reflect off the mirrors before it never hits either mirror surface again?

b) What is the angle that the laser beam forms with the mirror surface when the beam hits a mirror for the last time?

Measuring angles approximately from an image is not sufficient as an answer.