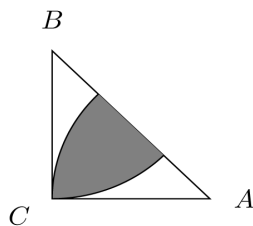# Finnish National High School Mathematics Contest

Open Division

October 9, 2008

1. The legs of an isosceles right triangle ABC have length 2 and AB is the hypotenuse. Two circles of radius 2 and centers A and B form together with AB the boundary of a figure. Determine the area of this figure.

2. The largest integer which is at most x is denoted by $\lfloor x \rfloor$. Solve the equation

$$x \cdot \lfloor x \rfloor = 910.$$

3. A circle has its centre at the origin, and it passes through the maximum and minimum points of the curve $y = x^3 + ax$. The circle is tangent to the curve at exactly two points. Determine a.

4. We call a set $\{a - d,\, a,\, a + d\}$ a *super triple*, if a is an integer and $d = 2^k$ for some natural number k.

a) Show that the set $\{0,\, 1,\, 2,\, \ldots,\, 2008\}$ has a subset A of 1340 elements such that A contains no super triples.

b) Show that every subset A of the set $\{0,\, 1,\, 2,\, \ldots,\, 2008\}$ with 1341 elements contains three consequtive integers.

Time allowed: **100 minutes**.

Write the solutions of different problems on different pages.

Write your name clearly.

Finnish National High School Mathematics Contest

Intermediate Division

October 9, 2008

1. The largest integer which is at most x is denoted by $\lfloor x \rfloor$. Solve the equation

$$x \cdot \lfloor x \rfloor = 50.$$

2. The surface of a qube is painted blue. Then the cube is cut by planes parallel to its faces into 27 pieces (not necessarily of the same size). Determine the proportion of the total surface area of the pieces which is painted blue.

3. Determine the largest integer which is a multiple of 11 and which has a decimal representation in which all the digits are different.

4. The vertices of a triangle are $A = (0, 0)$, $B = (0, 6)$ and $C = (6a, 6b)$, where $b \neq 0$. The medians of the triangle intersect at M, its circumscribed circle has center O and the altitudes intersect at H. Determine the ratio $MH : MO$.

Time allowed: **100 minutes**.

Write the solutions of different problems on different pages.

Write your name clearly.

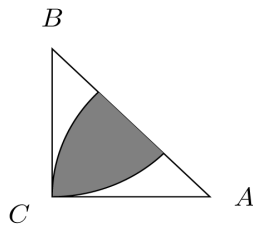# Finnish National High School Mathematics Contest

Basic Division

October 9, 2008

1. The surface of a $3 \times 3 \times 3$ qube is painted red. Then the cube is cut into 27 small qubes of equal size. Determine the proportion of the total surface area of the 27 small qubes which is painted red.

2. The *factorial* of a positive integer n is the product $1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$. The factorial of n is denoted by $n!$. For example, $3! = 1 \cdot 2 \cdot 3 = 6$. Solve N from the equation

$$6! \cdot 7! = N!.$$

3. The legs of an isosceles right triangle ABC have length 2 and AB is the hypotenuse. Two circles of radius 2 and centers A and B form together with AB the boundary of a figure. Determine the area of this figure.

4. Solve the equation

$$\sqrt{17 + x - 8\sqrt{x+1}} + \sqrt{5 + x - 4\sqrt{x+1}} = 6.$$

Time allowed: **100 minutes**.

Write the solutions of different problems on different pages.

Write your name clearly.