High School Mathematics Competition 2010

First Round, Basic Level

The first six problems are multiple choice problems. Each question has zero to four correct answers. Please answer on the designated form by writing a "+" if you think the alternative is correct, a "−", if you think the alternative is incorrect. Each correct answer is awarded by one point, incorrect or unintelligible answers are worth 0 points. Problems 7 and 8 are of the traditional type. They should be answered on a separate sheet. These problems are worth 6 points each.

Do not forget to fill in your name, school, home address and email address on the answer form. Writ your name clearly on the answer sheet of problems 7 and 8, too.

Time allowed: **120 minutes**.

1. The lengths of two sides of a triangle are 3 and 4. It is possible that the area A of the triangle satisfies:

a) $A < 1$ b) $A = 3$ c) $A = 6$ d) $A > 7$

2. It is possible to write the polynomial $x^4 - 1$ as the product

a) $(x^2 - 1)(x^2 + 1)$ b) $(x - 1)(x + 1)(x^2 + 1)$
c) $(x - 1)(x^3 + x^2 + x + 1)$ d) $(x + 1)(x^3 - x^2 + x - 1)$

3. The radius of the base of a right circular cylinder equals r and the height of the cylinder is h. The volume of the cylinder is 1 and its total surface area is 12. Then $1/r + 1/h$ equals

a) 1 b) 3 c) 4 d) 12

4. A positive integer is a *prime number*, if it has exactly two positive factors. The smallest prime number p for which $2010 \cdot p$ is the square of an integer is

a) 3 b) 5
c) some other number d) there is no such number

5. A cubic polynomial equation with integer coefficients can have

a) 0 b) 1 c) 2 d) 3 solutions

6. How many different values can the expression

$$\frac{x}{|x|} + \frac{y}{|y|} + \frac{z}{|z|} + \frac{xyz}{|xyz|}$$

have for different non-zero real numbers x, y and z?

a) at most 3 b) at least 3, but at most 6
c) more than 6, but a finite number d) infinitely many values

7. The areas of the faces of a rectangular parallelepiped are 6, 8 and 12 units of area. Determine the volume of the parallelepiped.

8. The sum of the digits of a four digit number is 16, the third digit is the sum of the first two digits and the second digit is two times the fourth digit. When the digits are written in the opposite order, a number 729 less than the original number is obtained. Determine the (original) number.

High School Mathematics Competition 2010

First Round, Intermediate Level

1. Matti and Kerkko decided to paint a fence. Working alone, Matti would have painted the fence in 3 hours and Kerkko in 4 hours. They start to paint at 12:00. After some time, a disagreement about the painting process arises and the boys quarrel for 10 minutes, during which time the job is at a standstill. On top of that, Kerkko loses his temper and leaves the job. Matti carries the job to an end at 14:25. When did the quarrel start?

2. The inscribed circle of a right triangle divides the hypotenuse in parts of length a and b. Show that the area of the triangle equals ab.

3. Determine the smallest positive integer for which $2664n$ is the square of an integer.

4. A sequence of numbers is said to be of the Fibonacci type, if every number in the sequence is the sum of the two immediately preceding numbers. Determine the fifth number in such a sequence, given that the tenth number is 322 and the numbers in the sequence are positive integers.

Time allowed: **120 minutes**.

Each answer on a different side of paper.

Write your name and contact information (school, home address and email address) clearly on the paper.

High School Mathematics Competition 2010

First Round, Open Level

1. The inscribed circle of a right triangle divides the hypotenuse in parts of length a and b. Show that the area of the triangle equals ab.

2. a, b and c are integers, $0 < a < b < c$. The average of the numbers a^{-1}, b^{-1}, c^{-1} and $\frac{1}{4}$ is $\frac{5}{16}$. Determine a, b and c.

3. x is an acute angle such that $\sin x$, $\sin(2x)$ and $\sin(4x)$ form an increasing arithmetic sequence. Determine the value of $\cos^3 x - \cos x$.

4. A knight is in the origin of an infinite chessboard. If the side of a square in the board is the unit of length, we can fix a coordinate system such that the centers of the squares have integer coordinates and the distances between the squares are measured from the center of one square to the center of another square. It is well-known that the knight can move from a square to any square at the distance $\sqrt{5}$ from the original square. Determine the smallest possible number of knight moves leading from the origin to a square at the distance $\sqrt{281}$ from the origin.

Time allowed: **120 minutes**.

Each answer on a different side of paper.

Write your name and contact information (school, home address and email address) clearly on the paper.